

The Tax Man

Name: _____

Class/Section: _____

The Problem

The Tax Man has placed the numbers 1 through 20 on the table. You may collect numbers for your score, but every choice comes with a tax. Your goal is to finish with a larger total than the Tax Man.

Rules

- On your turn, choose one unclaimed number.
- The Tax Man immediately collects every **unclaimed proper factor** of your number. A proper factor divides the number exactly but is smaller than the number itself.
- Your move is legal only when the Tax Man can collect at least one number.
- Continue until no legal move remains. The Tax Man then collects every number still on the table.
- Add your numbers. Add the Tax Man's numbers. The greater total wins.

Example. Suppose you choose 12 first. You collect 12, while the Tax Man collects

1, 2, 3, 4, 6.

Later in the game, the Tax Man receives only those proper factors of 12 that are still unclaimed.

Warm-Up Game: Numbers 1–12

Cross out each number as it is collected. Circle your choices and place a small *T* beside the Tax Man's numbers.

1

2

3

4

5

6

7

8

9

10

11

12

Turn	Your pick	Numbers collected by the Tax Man	Running score
1			
2			
3			
4			
5			

Your final score:

Tax Man's final score:

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MAIN CHALLENGE – ATTEMPT 1

Name: _____

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Numbers 1–20

Play a complete game. Record the tax paid on every turn so another person can reconstruct your game.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Turn	Your pick	Numbers collected by the Tax Man	Running score
1			
2			
3			
4			
5			
6			
7			

Your numbers	Tax Man's numbers	Numbers left at the end

Your final score:

Tax Man's final score:

Notice and wonder.

What did you notice about strong moves, expensive moves, prime numbers, or numbers that became impossible to choose?

One change I will make in my next attempt:

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MAIN CHALLENGE – ATTEMPT 2

Name: _____

Class/Section: _____

Try to Improve Your Score

Use what you learned from the first game. You may discuss strategy, but your recorded sequence must be complete and legal.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Turn	Your pick	Numbers collected by the Tax Man	Running score
1			
2			
3			
4			
5			
6			
7			

Your final score:

Tax Man’s final score:

Make Your Strategy Visible

1. Which move gave you the best value? Explain using the score gained and the tax paid.
2. Which available number did you deliberately avoid? Why?
3. How did the order of your choices affect later moves?

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INVESTIGATE, CONJECTURE, JUSTIFY

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From a Game to a Mathematical Investigation

Use examples, factor relationships, diagrams, tables, or a carefully organized argument. A strong response explains *why*, not only what happened in one game.

1. The total of all numbers from 1 through 20 is fixed. Find that total without adding the numbers one at a time. Explain your method.
2. What must your score be to beat the Tax Man? Explain how you know.
3. Investigate prime numbers in this game. When can a prime be chosen? When does a prime become trapped for the Tax Man?
4. Compare two possible first moves. Which is stronger, and what evidence supports your decision?
5. Find your best legal sequence for the numbers 1 through 20. Record the sequence clearly and justify at least three key decisions.
6. Generalize the game to the numbers 1 through n . What features of n might affect the best strategy?

Final conjecture.

State your strongest claim about how to play Tax Man well. Include evidence and identify anything you have not yet proved.