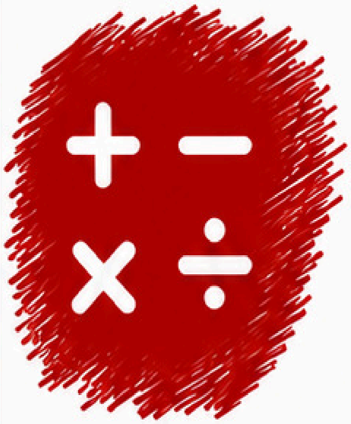


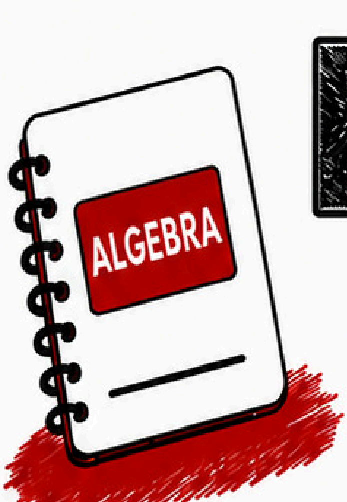
PAN AMERICAN SCHOOL OF BAHIA

GRADE 9

MATH HL

ALGEBRA

NOTE PACKAGE



f(x), g(x), a_n, $\log_a x$, sin θ,...

Name: ______________________

A, B, C, D, ...

Prepared by Chad Williams

Algebra Learning Objectives

Conceptual Goals

- Students will develop a deep understanding of algebra skills, including proper rounding techniques, exponent rules, and solving equations and inequalities involving a single variable.
- Students will connect and apply these foundational skills to more complex problems and real-world situations.

Competency Goals

- Apply rounding skills to express values to various degrees of accuracy.
- Solve equations and inequalities and analyze the meaning of solutions in the original context.
- State and apply exponent rules to simplify expressions.
- Model real-world situations with linear equations and inequalities and explain the meaning of solutions.

Character Goals

- Develop curiosity about numbers, quantities, algebra, and mathematical concepts.
- Build an inquisitive mindset by actively seeking the “why” and “how” behind mathematical relationships.

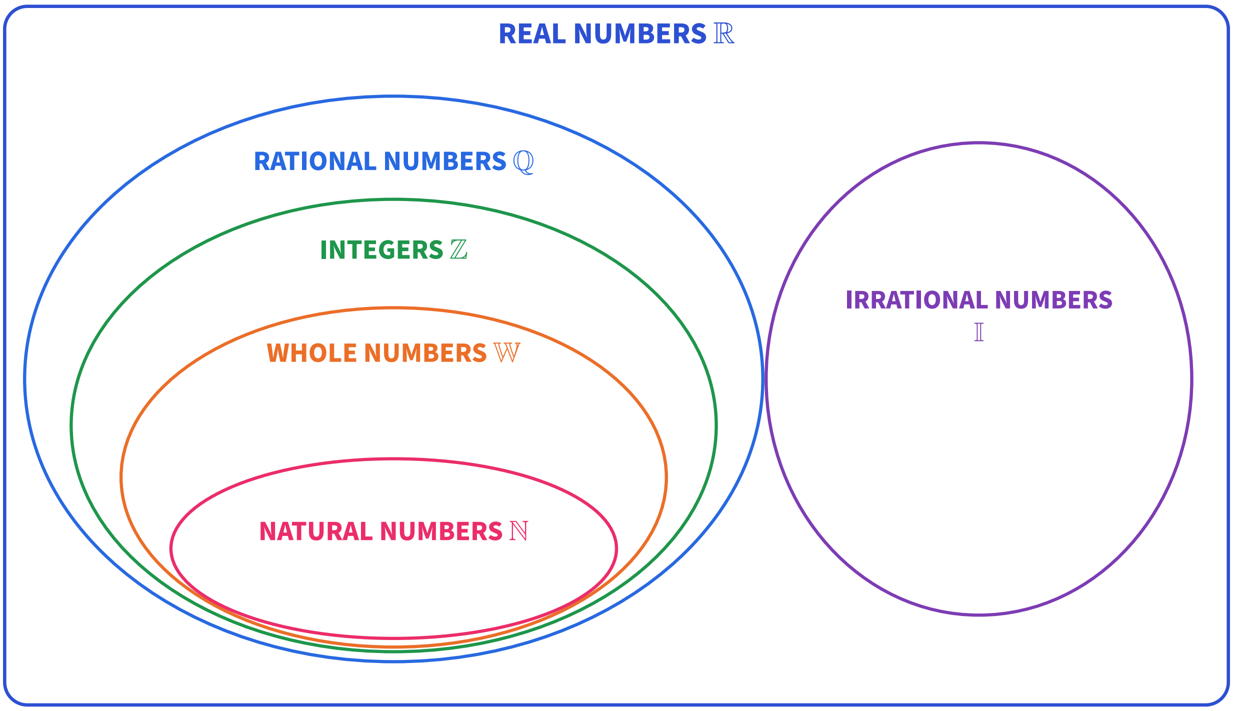
Lines of Inquiry

- How can I use mathematical rules and operations to simplify, solve, and represent expressions and equations accurately?
- What strategies help me make precise and meaningful decisions when rounding or interpreting numerical values?
- How do equations and inequalities help me represent and solve real-world problems?
- Why is it important to communicate solutions in the correct form, and how can I justify each step?

Unit 1: Algebra Learning Objectives

Math Concept	I Can Statement
Number System	I can classify, compare, and order rational and irrational numbers.
Rounding & Accuracy	I can round values to a specified degree of accuracy and determine upper and lower bounds.
Scientific Notation	I can convert, compare, and order numbers written in scientific notation.
Exponent Laws	I can apply exponent laws to simplify expressions involving integer exponents.
Fractional Exponents	I can convert between radical notation and fractional exponents.
Radical Expressions	I can simplify radicals and identify equivalent radical forms.
Operations with Radicals	I can add, subtract, multiply, and divide radical expressions.
Rationalizing Denominators	I can rationalize denominators using radicals and conjugates.
Solving Equations	I can solve one-variable equations and justify each step in my solution.
Solving Inequalities	I can solve and graph one-variable inequalities on a number line.
Compound Inequalities	I can solve and graph compound inequalities involving "and" and "or."
Absolute Value	I can solve and graph absolute value equations and inequalities.
Real-World Modeling	I can represent and solve real-world problems using equations and inequalities.

The Number System



Natural Numbers	**Counting numbers beginning at 1:** $1, 2, 3, 4, \ldots$
Whole Numbers	Natural numbers together with **zero**: $0, 1, 2, 3, 4, \ldots$
Integers	Whole numbers and their **negatives**: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Rational Numbers	Numbers that can be written as a fraction of two integers. This includes terminating and repeating decimals.
Irrational Numbers	Numbers that cannot be written as a fraction. Their decimals are non-terminating and non-repeating.

Classifying Number Sets

Number	Natural $\mathbb{N}$	Whole $\mathbb{W}$	Integer $\mathbb{Z}$	Rational $\mathbb{Q}$
3				
3.222				
0				
-4.5				
$-\frac{3}{5}$				
54				
-5				
$\frac{23}{3}$				
0.667				
$-1{,}364{,}698$				

Determine whether each statement is true or false. Justify your answer using definitions and/or examples.

1 All whole numbers are rational numbers.

☐ True ☐ False

Justification:

2 All rational numbers are whole numbers.

☐ True ☐ False

Justification:

3 All rational numbers are integers.

☐ True ☐ False

Justification:

4 All integers are rational numbers.

☐ True ☐ False

Justification:

5 All whole numbers are integers.

☐ True ☐ False

Justification:

6 All integers are whole numbers.

☐ True ☐ False

Justification:

Significant Figures

Significant Figures	**Digits that count when determining the precision of a number.**
Insignificant Figures	**Digits that do not affect the precision of a number.**

Examples

$\mathbf{25}$	$0.00\mathbf{73}$	$\mathbf{4.05}$	$\mathbf{10.20}$	$\mathbf{100}.$
2 s.f.	**2 s.f.**	**3 s.f.**	**4 s.f.**	**3 s.f.**
$0.00\mathbf{250}$	$\mathbf{3} \times 10^3$	$\mathbf{3000}.$	$\mathbf{3.00} \times 10^4$	$0.0\mathbf{700}$
3 s.f.	**1 s.f.**	**4 s.f.**	**3 s.f.**	**3 s.f.**

Rule 1: All nonzero digits are significant.

Number	Significant Figures	# of Significant Figures	Nonsignificant Figures
482			
9173			
26.4			

Rule 2: Zeros between nonzero digits are significant.

Number	Significant Figures	# of Significant Figures	Nonsignificant Figures
304			
1002			
50.09			

Significant Figures – Rules 3 to 5

Rule 3: Trailing zeros in a decimal are significant.

Number	Significant Figures	# of Significant Figures	Nonsignificant Figures
45.00			
7.8900			
120.00			

Rule 4: Zeros to the left of the first nonzero digit are not significant.

Number	Significant Figures	# of Significant Figures	Nonsignificant Figures
0.0042			
0.091			
0.000701			

Rule 5: Trailing zeros in an integer may or may not be significant because of rounding.

Number	Significant Figures	# of Significant Figures	Nonsignificant Figures
4000			
1200			
50 000			

Count the Significant Figures

1. 482
2. 0.0042
3. 45.00
4. 1200
5. 7.8900
6. 50.09
7. 0.091
8. 304
9. 600.0
10. 0.005060
11. $90\,040$
12. 0.000800

Round to Significant Figures

1. Round 78.321 to **3 significant figures**.
2. Round 0.009541 to **2 significant figures**.
3. Round 14.0408 to **5 significant figures**.
4. Round 28.702 to **4 significant figures**.
5. Round 0.03468 to **3 significant figures**.
6. Round 0.76714 to **2 significant figures**.
7. Round 456.9871 to **4 significant figures**.
8. Round 0.00070182 to **2 significant figures**.
9. Round 905.067 to **4 significant figures**.
10. Round 0.00049873 to **3 significant figures**.
11. Round $67\,849$ to **2 significant figures**.
12. Round 9.9951 to **3 significant figures**.

Rounding Numbers

Place Value											
Millions			Thousands			Ones			Decimals		
Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
2	8	6	4	0	1	5	2	7 .	3	7	9

Rounding Rules

Decimal Places

Round to a specified number of digits after the decimal point. For example, $4.7864 \approx 4.79$ to 2 decimal places.

Significant Figures

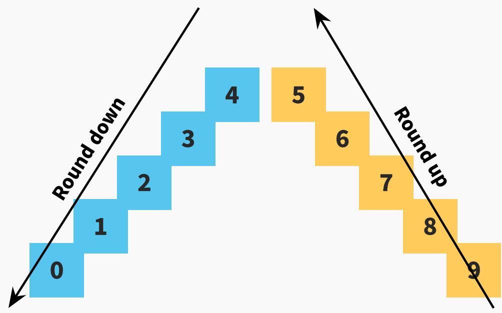
Begin at the first nonzero digit and keep the required number of meaningful digits. For example, $0.0054720 \approx 0.0055$ to 2 significant figures.

Nearest Whole Number, Ten, or Hundred

Use place value to identify the rounding digit, then examine the digit immediately to its right. For example, $347 \approx 350$ to the nearest ten.

Rounding Numbers

Look one digit to the right.



0, 1, 2, 3, 4: Keep the rounding digit the same.

5, 6, 7, 8, 9: Increase the rounding digit by 1.

Worked Examples: Significant Figures

a Round $62\,748$ to **2 significant figures**.

The first two significant digits are 6 and 2.

The next digit is 7, so round 2 up to 3.

$$62\,748 \approx \boxed{63\,000}$$

b Round 0.004936 to **3 significant figures**.

The first three significant digits are 4, 9, and 3.

The next digit is 6, so round 3 up to 4.

$$0.004936 \approx \boxed{0.00494}$$

Quick Check

Nearest centimetre

73.46 cm

___

2 decimal places

18.3764

___

2 significant figures

0.004862

___

Rounding Practice

1. Round each number to the nearest 10.

a 74	**b** 46	**c** 135	**d** 289
e 503	**f** 876	**g** 2 994	**h** 18 647

2. Round each number to the nearest 100.

a 348	**b** 762	**c** 1 249	**d** 3 851
e 6 405	**f** 9 950	**g** 27 649	**h** 84 372

3. Round each number to the nearest 1000.

a 1 482	**b** 3 607	**c** 8 499	**d** 12 650
e 25 320	**f** 49 780	**g** 106 499	**h** 382 615

4. Round each number to 1 decimal place.

a 4.67	**b** 8.12	**c** 19.95	**d** 32.04
e 0.786	**f** 6.349	**g** 47.052	**h** 105.978

5. Round each number to 2 decimal places.

a 3.746	**b** 0.285	**c** 9.103	**d** 14.997
e 7.0648	**f** 26.1351	**g** 0.0947	**h** 213.6086

6. Round each number to 3 decimal places.

a 2.4876	**b** 9.1034	**c** 0.7568	**d** 5.9995
e 31.0847	**f** 72.6392	**g** 6.20549	**h** 0.04086

7. Each number below has already been rounded. Write one possible original number.

a	**Nearest 10** 340 __________	**b**	**Nearest 100** 2 700 __________	**c**	**Nearest 1000** 18 000 __________	**d**	**Nearest 10** 6 420 __________
e	**1 decimal place** 7.3 __________	**f**	**2 decimal places** 4.86 __________	**g**	**3 decimal places** 0.125 __________	**h**	**2 decimal places** 19.40 __________

Scientific Notation

Scientific Notation	Writing a number as a value between 1 and 10 multiplied by a power of 10.
Standard Form	The usual way of writing a number using digits rather than powers of 10.

A Very Large Number

The mass of the Sun is approximately

$$1.988 \times 10^{30}\ \textbf{kg}$$

Scientific notation is more efficient than writing all 31 digits.

A Very Small Number

The mass of a uranium atom is approximately

$$3.95 \times 10^{-22}\ \textbf{g}$$

Scientific notation makes extremely small values easier to read.

Remember: How Scientific Notation Is Written

A number written in scientific notation has the form

$$\boxed{N = a \times 10^n}$$

where

$$\boxed{1 \leq |a| < 10} \quad \text{and} \quad \boxed{n \in \mathbb{Z}}.$$

- The coefficient a has exactly one nonzero digit to the left of the decimal point.
- The exponent n tells how many places the decimal point moves.
- A positive exponent moves the decimal point to the right when converting to standard form.
- A negative exponent moves the decimal point to the left when converting to standard form.
- An exponent of zero does not change the coefficient because $10^0 = 1$.

Indicate whether each number is written in scientific notation. Explain your reasoning.

Number	Scientific Notation?	Explain
3.2×10^4		
0.0025×10^6		
7.91×10^{-3}		
23×10^2		

Scientific Notation Practice

Scientific notation	Large numbers	Small numbers
$a \times 10^n$	Positive exponent	Negative exponent
$1 \leq a < 10$	$4.5 \times 10^6 = 4\,500\,000$	$5.92 \times 10^{-4} = 0.000592$

Write in scientific notation.

1. $6\,300\,000$
2. 0.00047
3. $85\,200$
4. 0.0000063
5. $9\,040$
6. $72\,000\,000$

Write in standard form.

1. 5.7×10^4
2. 8.2×10^{-3}
3. 1.36×10^7
4. 4.09×10^{-5}
5. 7.005×10^2
6. 9.4×10^{-7}

Order each set of numbers from least to greatest.

1. $4.6 \times 10^3, \quad 8\,200, \quad 2.9 \times 10^3, \quad 5\,400$
2. $7.8 \times 10^{-2}, \quad 0.054, \quad 6.2 \times 10^{-2}, \quad 0.091$
3. $0.00048, \quad 6.5 \times 10^{-5}, \quad 3.2 \times 10^{-4}, \quad 0.0017$
4. $3.1 \times 10^2, \quad 175, \quad 8.9 \times 10^1, \quad 406$
5. $7.2 \times 10^5, \quad 6.8 \times 10^6, \quad 9.5 \times 10^4, \quad 4.1 \times 10^7$
6. $3.6 \times 10^{-3}, \quad 0.00091, \quad 7.4 \times 10^{-4}, \quad 0.0052$

Scientific Notation – Extra Practice

A. Convert to Scientific Notation

Write each number in scientific notation.

1. $58\,000$
2. 0.00073
3. $406\,000\,000$
4. 0.00285
5. $9\,030\,000$
6. 0.000000064

B. Convert to Standard Form

Write each number in standard decimal form.

1. 3.7×10^{5}
2. 6.42×10^{-3}
3. 8.005×10^{2}
4. 2.9×10^{-6}
5. 1.074×10^{7}
6. 5.63×10^{0}

C. Order the Numbers

Write each list from **least to greatest**. You may convert the numbers to the same form first.

1. 3.8×10^{3}, $4\,200$, 7.5×10^{2}, $36\,000$, 9.1×10^{1}, 5.4×10^{4}
2. 0.0062, 8.4×10^{-3}, 0.00075, 4.1×10^{-2}, 9.6×10^{-4}, 0.015
3. 3.06×10^{5}, $306\,500$, 3.005×10^{5}, $305\,900$, 3.1×10^{4}, 3.061×10^{5}
4. 7.4×10^{-5}, 0.00081, 6.9×10^{-4}, 0.000072, 1.2×10^{-3}, 9.5×10^{-5}

Limits of Accuracy

An exact value may be slightly greater or smaller than a rounded measurement. The **lower bound** and **upper bound** describe the range of possible exact values.

Lower and Upper Bounds

Lower bound: the smallest value that would round to the stated value.

Upper bound: the smallest value that would round to the next possible rounded value.

Interval Rule

Write the exact value between the lower and upper bounds:

$$\boxed{\text{Lower bound} \leq x < \text{Upper bound}}$$

The lower bound is included, but the upper bound is not included.

Example: Rounded to the Nearest Whole Number

If $x = 18$, correct to the nearest whole number, then the rounding unit is 1.

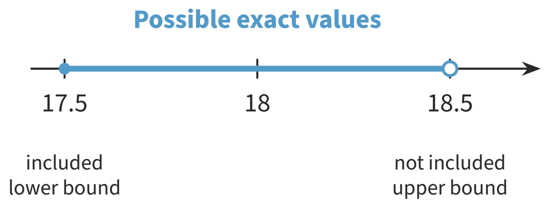
$$\frac{1}{2} = 0.5$$

$$18 - 0.5 = 17.5$$

$$18 + 0.5 = 18.5$$

Therefore,

$$\boxed{17.5 \leq x < 18.5}$$



Find the Lower and Upper Bounds

Write the lower and upper bound for each rounded value.

1. 24, nearest whole number

Lower bound: ____________________

Upper bound: ____________________

2. 57, nearest whole number

Lower bound: ____________________

Upper bound: ____________________

3. 8.3, nearest tenth

Lower bound: ____________________

Upper bound: ____________________

4. 14.7, nearest tenth

Lower bound: ____________________

Upper bound: ____________________

5. 6.42, nearest hundredth

Lower bound: ____________________

Upper bound: ____________________

6. 19.85, nearest hundredth

Lower bound: ____________________

Upper bound: ____________________

7. 120, nearest ten

Lower bound: ____________________

Upper bound: ____________________

8. 3500, nearest hundred

Lower bound: ____________________

Upper bound: ____________________

Measurement Bounds

Find the range of possible exact values for each measurement. Write each answer in the form

lower bound $\leq x <$ upper bound.

1. A length is measured as 12 cm, rounded to the nearest centimetre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

2. A mass is measured as 4.8 kg, rounded to the nearest tenth of a kilogram.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

3. A distance is measured as 36.25 m, rounded to the nearest hundredth of a metre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

4. A time is measured as 9.6 s, rounded to the nearest tenth of a second.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

5. A road is measured as 250 m, rounded to the nearest ten metres.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

6. The volume of a container is measured as 3.47 L, rounded to the nearest hundredth of a litre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

7. A package has a mass of 680 g, rounded to the nearest ten grams.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

8. A running route is measured as 5.3 km, rounded to the nearest tenth of a kilometre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

9. The height of a building is measured as 124 m, rounded to the nearest metre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

10. A liquid has a volume of 0.625 L, rounded to the nearest thousandth of a litre.

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq x < \_\_\_\_\_\_\_\_\_\_\_\_$$

Perimeter Bounds

Use the lower and upper bounds of each side length to find the **smallest** and **largest** possible perimeter.

1. A square has side length 8 cm, rounded to the nearest centimetre.

Lower bound of side $=$ ____________________

Upper bound of side $=$ ____________________

Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

2. A rectangle has length 12 m and width 7 m, both rounded to the nearest metre.

Smallest perimeter $=$ ____________________

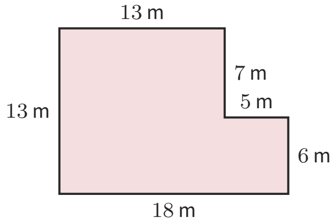
Largest perimeter $<$ ____________________

3. A triangle has side lengths 5.4 cm, 7.2 cm, and 9.1 cm, each rounded to the nearest tenth of a centimetre.

Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

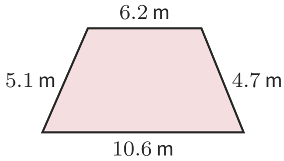
4. The side lengths are rounded to the nearest metre.



Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

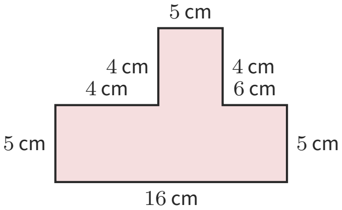
5. The side lengths are rounded to the nearest tenth of a metre.



Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

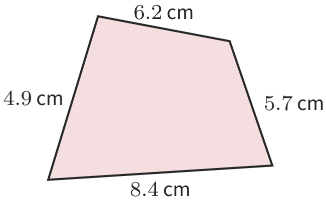
6. The side lengths are rounded to the nearest centimetre.



Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

7. The side lengths are rounded to the nearest tenth of a centimetre.



Smallest perimeter $=$ ____________________

Largest perimeter $<$ ____________________

Area Bounds

Find the smallest and largest possible area. Use the lower bounds of the dimensions for the smallest area and the upper bounds for the largest area.

1. A rectangle has length 10 cm and width 6 cm, both rounded to the nearest centimetre.

Smallest area $=$ ______________________ cm^2

Largest area $<$ ______________________ cm^2

2. A rectangle has length 8.4 m and width 3.2 m, both rounded to the nearest tenth of a metre.

Smallest area $=$ ______________________ m^2

Largest area $<$ ______________________ m^2

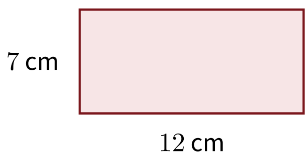
3. A square has side length 15 cm, rounded to the nearest centimetre.

Smallest area $=$ ______________________ cm^2

Largest area $<$ ______________________ cm^2

Area Bounds from Diagrams

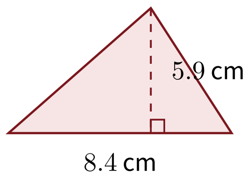
4 **The side lengths are rounded to the nearest centimetre.**



Smallest area $=$ ________________ cm^2

Largest area $<$ ________________ cm^2

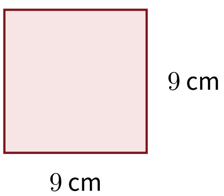
5 **The base and perpendicular height are rounded to the nearest tenth of a centimetre.**



Smallest area $=$ ________________ cm^2

Largest area $<$ ________________ cm^2

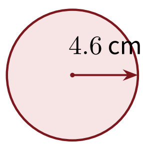
6 **The side length is rounded to the nearest centimetre.**



Smallest area $=$ ________________ cm^2

Largest area $<$ ________________ cm^2

7 **The radius is rounded to the nearest tenth of a centimetre. Use $A = \pi r^2$.**



Smallest area $=$ ________________ cm^2

Largest area $<$ ________________ cm^2

Bounds – Word Problems

Use lower and upper bounds to solve each problem. Show all calculations and include the correct units in each answer.

1. A rectangular garden is measured as 14 m by 9 m. Both measurements are rounded to the nearest metre.

a) Find the lower and upper bounds for the length.

b) Find the lower and upper bounds for the width.

c) Find the smallest possible perimeter.

d) Find the largest possible perimeter.

2. A square tile has a side length of 6.5 cm, rounded to the nearest tenth of a centimetre.

a) Find the lower and upper bounds for the side length.

b) Find the smallest possible area.

c) Find the largest possible area.

3. A triangular field has side lengths 24 m, 31 m, and 42 m. Each measurement is rounded to the nearest metre.

a) Find the smallest possible perimeter.

b) Find the largest possible perimeter.

Bounds – Additional Practice

Use lower and upper bounds to solve each problem. Show all calculations and include the correct units.

4. A square courtyard has a measured side length of 11 m, rounded to the nearest metre.

a) Find the lower and upper bounds for the side length.

b) Find the smallest and largest possible perimeter.

5. A rectangular lawn is measured as 9 m long and 6 m wide. Both measurements are rounded to the nearest metre.

a) Find the lower and upper bounds for the length and width.

b) Find the smallest and largest possible perimeter.

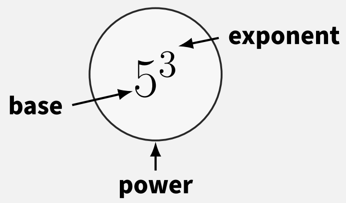
6. A triangular patio has measured side lengths of 9 m, 11 m, and 13 m. Each measurement is rounded to the nearest metre.

a) Find the lower and upper bounds for the longest side.

b) Find the smallest and largest possible perimeter.

Rules of Exponents

Language of Powers



$\boxed{\text{base}^{\text{exponent}}}$

In 5^3, the base is 5, the exponent is 3, and the power is the complete expression 5^3.

$$5^3 = 5 \times 5 \times 5 = 125$$

Rule	Formula	Meaning	Example
1. Zero Exponent	$a^0 = 1$	Any nonzero number or expression raised to the power of 0 is equal to 1.	$7^0 = 1$
2. Product Rule	$a^m \cdot a^n = a^{m+n}$	When multiplying powers with the same base, keep the base and add the exponents.	$x^3 \cdot x^5 = x^8$
3. Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$	When dividing powers with the same base, keep the base and subtract the exponents.	$\frac{y^9}{y^4} = y^5$
4. Power to a Power	$(a^m)^n = a^{mn}$	When raising a power to another power, multiply the exponents.	$(m^3)^4 = m^{12}$
5. Product to a Power	$(ab)^m = a^m b^m$	When raising a product to a power, apply the exponent to every factor inside the brackets.	$(2x)^3 = 2^3 x^3 = 8x^3$
6. Quotient to a Power	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	When raising a quotient to a power, apply the exponent to both the numerator and denominator.	$\left(\frac{x}{3}\right)^2 = \frac{x^2}{9}$
7. Fractional Exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	The denominator gives the root, and the numerator gives the power.	$16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 8$
8. Negative Exponent	$a^{-n} = \frac{1}{a^n}$	A negative exponent means take the reciprocal of the base and change the exponent to positive.	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Important Conditions

- For the zero-exponent rule, the base must be nonzero: $a \neq 0$.
- A denominator can never equal zero.
- The product and quotient rules apply only when the bases are the same.
- A negative exponent does not make the value negative; it indicates a reciprocal.

Product of Powers $a^m \cdot a^n = a^{m+n}$

Simplify each expression. Write your answer using a single power.

1. $x^3 \cdot x^5$
2. $y^7 \cdot y^2$
3. $a^4 \cdot a^6$
4. $m^{11} \cdot m^3$
5. $p^9 \cdot p$
6. $u^5 \cdot u^5$
7. $k^2 \cdot k^8$
8. $q^7 \cdot q^4$
9. $b^{12} \cdot b^3$
10. $r \cdot r^9$
11. $x^6 \cdot x^4$
12. $y^9 \cdot y^3$
13. $a^8 \cdot a^5$
14. $m^2 \cdot m^{13}$
15. $p^{10} \cdot p^7$
16. $k^{15} \cdot k$
17. $q^{11} \cdot q^6$
18. $b^5 \cdot b^{14}$
19. $r^{12} \cdot r^8$
20. $t^3 \cdot t^{17}$
21. $z^9 \cdot z^9$
22. $w \cdot w^{16}$
23. $n^{20} \cdot n^4$
24. $c^7 \cdot c^2$
25. $h^{13} \cdot h^7$
26. $d^{18} \cdot d$
27. $v^6 \cdot v^{15}$

Quotient of Powers $\dfrac{a^m}{a^n} = a^{m-n}$

Simplify each expression. Write each answer using positive exponents.

1. $\dfrac{x^9}{x^2}$

2. $\dfrac{y^{11}}{y^5}$

3. $\dfrac{a^6}{a^4}$

4. $\dfrac{m^{10}}{m^3}$

5. $\dfrac{p^8}{p}$

6. $\dfrac{u^7}{u^7}$

7. $\dfrac{k^{12}}{k^9}$

8. $\dfrac{q^5}{q^2}$

9. $\dfrac{r^{13}}{r^{11}}$

10. $\dfrac{x^{14}}{x^6}$

11. $\dfrac{y^{12}}{y^3}$

12. $\dfrac{a^9}{a^2}$

13. $\dfrac{m^{15}}{m^5}$

14. $\dfrac{p^{11}}{p^4}$

15. $\dfrac{k^8}{k}$

16. $\dfrac{q^{18}}{q^7}$

17. $\dfrac{r^{16}}{r^9}$

18. $\dfrac{t^{21}}{t^3}$

19. $\dfrac{z^{19}}{z^{10}}$

20. $\dfrac{w^{17}}{w^2}$

21. $\dfrac{n^{24}}{n^{12}}$

22. $\dfrac{c^{13}}{c^6}$

23. $\dfrac{h^{15}}{h^4}$

24. $\dfrac{d^{22}}{d^{11}}$

Power of a Power $(a^m)^n = a^{mn}$

Simplify each expression. Write your answer using a single power.

1. $(x^2)^4$
2. $(y^3)^5$
3. $(a^4)^2$
4. $(m^5)^3$
5. $(p^6)^2$
6. $(u^2)^7$
7. $(k^3)^4$
8. $(q^5)^2$
9. $(b^7)^2$
10. $(r^4)^3$
11. $(x^3)^3$
12. $(y^2)^6$
13. $(a^5)^4$
14. $(m^7)^2$
15. $(p^3)^5$
16. $(k^4)^3$
17. $(q^6)^4$
18. $(b^3)^8$
19. $(r^5)^6$
20. $(t^2)^{11}$
21. $(z^7)^3$
22. $(w^4)^5$
23. $(n^3)^9$
24. $(c^8)^2$
25. $(h^2)^{12}$
26. $(d^5)^7$
27. $(x^6)^2$
28. $(y^4)^4$
29. $(a^9)^3$
30. $(m^6)^5$

Power of a Product $(ab)^n = a^n b^n$

Simplify each expression. Evaluate the numerical coefficient and write all variables using positive exponents.

1. $(2x)^3$

2. $(3y^2)^2$

3. $(5a)^4$

4. $(2m^3)^3$

5. $(4p^2)^2$

6. $(6u)^3$

7. $(3k^2)^3$

8. $(2q)^5$

9. $(7b^3)^2$

10. $(5r)^4$

11. $(4x^2)^3$

12. $(2y^3)^2$

13. $(3a^2)^4$

14. $(5m)^3$

15. $(2p^4)^2$

16. $(7k^2)^2$

17. $(6q^2)^3$

18. $(4b)^5$

19. $(3r^3)^2$

20. $(8t)^4$

21. $(5z^2)^3$

22. $(2w^4)^2$

23. $(9n)^2$

24. $(7c^3)^3$

25. $(4h^2)^4$

26. $(6d)^5$

27. $(3v^4)^3$

Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Simplify each expression. Evaluate the numerical denominator and write all variables using positive exponents.

1. $\left(\frac{x}{2}\right)^3$

2. $\left(\frac{y^2}{3}\right)^2$

3. $\left(\frac{a}{5}\right)^4$

4. $\left(\frac{m^3}{2}\right)^3$

5. $\left(\frac{p^2}{4}\right)^2$

6. $\left(\frac{u}{6}\right)^3$

7. $\left(\frac{k^2}{3}\right)^3$

8. $\left(\frac{q}{2}\right)^5$

9. $\left(\frac{b^3}{7}\right)^2$

10. $\left(\frac{r}{5}\right)^4$

11. $\left(\frac{x^2}{3}\right)^4$

12. $\left(\frac{y}{5}\right)^3$

13. $\left(\frac{a^3}{2}\right)^2$

14. $\left(\frac{m^2}{7}\right)^3$

15. $\left(\frac{p^4}{3}\right)^2$

16. $\left(\frac{k}{4}\right)^5$

17. $\left(\frac{q^3}{2}\right)^4$

18. $\left(\frac{b}{9}\right)^3$

19. $\left(\frac{r^2}{5}\right)^5$

20. $\left(\frac{t}{8}\right)^4$

21. $\left(\frac{z^5}{3}\right)^2$

22. $\left(\frac{w^2}{7}\right)^3$

23. $\left(\frac{n^4}{2}\right)^2$

24. $\left(\frac{c}{6}\right)^5$

25. $\left(\frac{h^3}{4}\right)^3$

26. $\left(\frac{d^2}{5}\right)^4$

27. $\left(\frac{v^4}{6}\right)^3$

Negative Exponents – Rewrite with Positive Exponents

Rewrite each expression using positive exponents only. Simplify where possible.

1. x^{-3}
2. y^{-5}
3. $a^{-2}b^{-3}$
4. $\frac{1}{m^{-4}}$
5. $p^{-1}q^{2}$
6. $\frac{u^3}{u^{-2}}$
7. $k^{-6} \cdot k^{2}$
8. $\frac{q^{-3}}{q^4}$
9. $\left(b^{-2}\right)^3$
10. $\left(\frac{r^{-1}}{s^{-2}}\right)^2$
11. $x^{-7}y^{2}$
12. $\frac{1}{a^{-3}b^2}$
13. $\frac{m^{-5}}{m^2}$
14. $\left(p^{-2}q^{-1}\right)^3$
15. $\frac{k^4}{k^{-3}}$
16. $\left(\frac{r^{-2}}{s^{3}}\right)^2$
17. $\frac{z^{-6}}{z^{-2}}$
18. $\left(t^{-3}\right)^4$
19. $\frac{1}{w^{-5}}$
20. $n^{-2} \cdot n^{11}$
21. $\frac{c^4}{c^{-7}}$
22. $\left(\frac{b^{-1}}{q^{2}}\right)^3$
23. $\frac{r^{-8}s^3}{r^{-2}}$
24. $\left(x^{-2}y^{-3}\right)^2$
25. $\frac{1}{h^{-2}d^3}$
26. $\left(\frac{m^{-1}}{n^{-2}}\right)^3$
27. $\frac{p^{-4}}{p^2}$
28. $\left(q^{-2}r^{3}\right)^{-1}$
29. $\frac{x^{-3}y^5}{x^2y^{-1}}$
30. $\left(\frac{a^{-2}b^{3}}{c^{-1}}\right)^2$

Fractional Exponents

Fractional Exponent	A power that has a fraction as its exponent. The denominator identifies the root, while the numerator identifies the power.

$$a^{m/n}$$

Take the nth root, then raise the result to the mth power.

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

a is the base, m is the power, and n is the root.

Examples

$$16^{1/2} = \sqrt{16} = 4$$

$$27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 9$$

$$81^{3/4} = \left(\sqrt[4]{81}\right)^3 = 27$$

$$8^{-2/3} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{4}$$

Simplify each expression. Write answers using positive exponents.

1. $16^{3/4}$

2. $81^{3/2}$

3. $125^{2/3}$

4. $\left(\frac{1}{32}\right)^{-3/5}$

5. $49^{1/2}$

6. $64^{2/3}$

7. $32^{3/5}$

8. $27^{-2/3}$

9. $16^{-3/4}$

10. $\left(\frac{64}{729}\right)^{1/3}$

11. $\left(\frac{81}{16}\right)^{3/4}$

12. $x^{1/2} \cdot x^{3/2}$

13. $\frac{x^{5/2}}{x^{1/2}}$

14. $\left(x^{2/3}\right)^3$

15. $\sqrt[3]{x^2}$

16. $x^{5/3}$

17. $4x^{1/2}$

18. $\left(x^{3/4}y^{1/2}\right)^2$

Matching – Fractional Exponents

Match each fractional-exponent expression with its value. Write the correct letter in the answer table.

1	$81^{1/2}$	**6**	$(-32)^{1/5}$	**11**	$\left(\frac{25}{16}\right)^{1/2}$
2	$64^{1/3}$	**7**	$625^{1/4}$	**12**	$\left(-\frac{8}{27}\right)^{1/3}$
3	$(-216)^{1/3}$	**8**	$343^{1/3}$	**13**	$\left(\frac{9}{25}\right)^{1/2}$
4	$16^{1/4}$	**9**	$(-64)^{1/3}$	**14**	$\left(-\frac{27}{8}\right)^{1/3}$
5	$243^{1/5}$	**10**	$1^{1/7}$	**15**	$\left(6\frac{1}{4}\right)^{1/2}$

A	$\frac{3}{5}$	**F**	7	**K**	9
B	-4	**G**	1	**L**	3
C	$\frac{5}{2}$	**H**	-6	**M**	$\frac{5}{4}$
D	2	**I**	5	**N**	-2
E	$-\frac{2}{3}$	**J**	$-\frac{3}{2}$	**O**	4

1	**2**	**3**	**4**	**5**
6	**7**	**8**	**9**	**10**
11	**12**	**13**	**14**	**15**

Fractional Exponents Practice

Instructions: Simplify each expression. Leave answers with positive exponents only.

1. $16^{\frac{1}{2}}$

2. $27^{\frac{1}{3}}$

3. $81^{\frac{3}{4}}$

4. $32^{\frac{2}{5}}$

5. $125^{\frac{2}{3}}$

6. $64^{\frac{5}{6}}$

7. $49^{-\frac{1}{2}}$

8. $8^{-\frac{2}{3}}$

9. $x^{\frac{1}{2}} \cdot x^{\frac{3}{2}}$

10. $\dfrac{y^{\frac{5}{2}}}{y^{\frac{1}{2}}}$

11. $\left(a^{\frac{2}{3}}\right)^3$

12. $(16x^4)^{\frac{1}{2}}$

13. $(27m^6)^{\frac{1}{3}}$

14. $\dfrac{32x^5}{x^{\frac{3}{2}}}$

15. $(81p^8q^4)^{\frac{1}{4}}$

16. $64^{-\frac{2}{3}}$

17. $\left(\dfrac{1}{16}\right)^{-\frac{3}{4}}$

18. $(9x^2y^4)^{\frac{1}{2}}$

19. $\dfrac{a^{\frac{7}{3}}}{a^{\frac{1}{3}}}$

20. $(125x^9)^{\frac{2}{3}}$

Exponent Rules – Mixed Practice

Simplify each expression. Write final answers with positive exponents.

1. $\dfrac{x^5 \cdot x^{-2}}{x^3}$

2. $\left(y^3 z^{-2}\right)^2 \cdot z^5$

3. $\dfrac{\left(a^4 b^{-3}\right)^2}{a^5}$

4. $\left(m^{-2} n^3\right)\left(m^4 n^{-1}\right)$

5. $\dfrac{\left(p^2 q^3\right)^2}{p^3 q}$

6. $\dfrac{\left(r^{-3} s^4\right)^3}{s^2}$

7. $\left(t^5 u^{-2}\right)\left(t^{-1} u^3\right)$

8. $\dfrac{\left(k^2 m^{-4}\right)^2}{k^{-1} m}$

9. $\left(x^{-2} y^3\right)^2\left(x^3 y^{-1}\right)$

10. $\dfrac{\left(a^{-3} b^2\right)\left(a^4 b^{-5}\right)}{a^{-1} b}$

11. $\dfrac{x^{-4} \cdot x^9}{x^2}$

12. $\left(y^{-1} z^4\right)^2 \cdot z^{-3}$

13. $\dfrac{\left(a^{-2} b^3\right)^3}{a^{-1}}$

14. $\left(m^5 n^{-2}\right)\left(m^{-3} n^6\right)$

15. $\dfrac{\left(p^{-1} q^2\right)^2}{p^3 q^{-1}}$

16. $\left(k^{-2} m^3\right)^2\left(k^5 m^{-1}\right)$

17. $\dfrac{\left(x^{-3} y^2\right)^2}{x^4 y^{-1}}$

18. $\left(z^2 w^{-3}\right)\left(z^{-5} w^6\right)$

19. $\dfrac{\left(n^3 c^{-2}\right)^2 \cdot c^5}{n^{-1}}$

20. $\left(\dfrac{a^{-2}}{b^3}\right)^2 \cdot a^5$

21. $\dfrac{\left(r^4 s^{-1}\right)^3}{r^{-2} s^5}$

22. $\left(p^{-3} q^2\right)\left(\dfrac{p^6}{q^{-1}}\right)$

23. $\dfrac{\left(c^{-2} d^4\right)^2}{c d^{-3}}$

24. $\left(\dfrac{u^3 v^{-2}}{u^{-1} v^4}\right)^2$

Radicals

Square Root	A value that, when multiplied by itself, equals the original number.
Radical	An expression containing a root symbol, such as $\sqrt{18}$.
Surd	An irrational number written exactly in radical form rather than as a decimal approximation.

Worked Example

Write $\sqrt{72}$ in simplest surd form.

$$\begin{aligned}\sqrt{72} &= \sqrt{36 \times 2} \\ &= \sqrt{36}\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

Choose the largest perfect-square factor whenever possible.

Write each expression in simplest surd form.

1. $\sqrt{12}$ **2.** $\sqrt{28}$ **3.** $\sqrt{45}$ **4.** $\sqrt{63}$

5. $\sqrt{72}$ **6.** $\sqrt{80}$ **7.** $\sqrt{98}$ **8.** $\sqrt{108}$

9. $\sqrt{125}$ **10.** $\sqrt{147}$ **11.** $\sqrt{162}$ **12.** $\sqrt{175}$

13. $\sqrt{192}$ **14.** $\sqrt{200}$ **15.** $\sqrt{242}$ **16.** $\sqrt{252}$

17. $\sqrt{288}$ **18.** $\sqrt{320}$ **19.** $\sqrt{245}$ **20.** $\sqrt{300}$

Simplifying Radicals with Variables

When simplifying variables under a square root, take out pairs of identical factors. Assume all variables are non-negative.

$$\sqrt{x^2} = x \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^5} = x^2\sqrt{x}$$

Variables

Worked Example

$$\sqrt{x^8} = x^4$$

Each pair of identical variables becomes one factor outside the radical.

Practice

Simplify each expression.

1. $\sqrt{x^2}$ **2.** $\sqrt{y^4}$ **3.** $\sqrt{z^6}$ **4.** $\sqrt{m^8}$ **5.** $\sqrt{n^{10}}$

Numbers with Variables

Worked Example

$$\begin{aligned}\sqrt{18x^2} &= \sqrt{9 \cdot 2}\sqrt{x^2} \\ &= 3x\sqrt{2}\end{aligned}$$

Practice

Simplify each expression.

6. $\sqrt{18x^2}$ **7.** $\sqrt{32y^2}$ **8.** $\sqrt{75m^2}$ **9.** $\sqrt{98a^2}$ **10.** $\sqrt{200b^2}$

Coefficients

Worked Example

$$3\sqrt{9} = 3(3) = 9$$

Practice

Simplify each expression.

11. $3\sqrt{9}$ **12.** $4\sqrt{25}$ **13.** $6\sqrt{16}$ **14.** $2\sqrt{49}$ **15.** $5\sqrt{36}$

Coefficients, Surds, and Variables

Worked Example

$$\begin{aligned}3\sqrt{8x^2} &= 3\sqrt{4 \cdot 2}\sqrt{x^2} \\ &= 3(2x\sqrt{2}) \\ &= 6x\sqrt{2}\end{aligned}$$

Practice

Simplify each expression.

16. $3\sqrt{2x^2}$ **17.** $2\sqrt{3y^2}$ **18.** $4\sqrt{5a^2}$

19. $3\sqrt{8m^2}$ **20.** $5\sqrt{12n^2}$

Simplifying Radicals with Variables and Mixed Practice

Simplify each expression.

1. $\sqrt{v^9}$

2. $\sqrt{x^2}$

3. $\sqrt{49y^2}$

4. $\sqrt{12a^2}$

5. $\sqrt{75m^2}$

6. $\sqrt{8n^2}$

7. $\sqrt{18p^4}$

8. $\sqrt{27b^2}$

9. $\sqrt{45r^2s^2}$

10. $\sqrt{32q^6}$

11. $2\sqrt{18}$

12. $3\sqrt{50}$

13. $5\sqrt{12}$

14. $4\sqrt{27}$

15. $6\sqrt{8}$

16. $7\sqrt{45}$

17. $2\sqrt{200}$

18. $9\sqrt{32}$

19. $3\sqrt{98}$

20. $8\sqrt{75}$

21. $2\sqrt{12x^2}$

22. $3\sqrt{50y^2}$

23. $4\sqrt{27a^2}$

24. $5\sqrt{8b^4}$

25. $\sqrt{32p^6}$

26. $6\sqrt{18m^2n^2}$

Operations with Radicals

Combine Like Radicals

Radicals can be added or subtracted only when they have the same simplified radicand.

$$3\sqrt{7} - 4\sqrt{7} = -\sqrt{7}$$

$$5\sqrt{3} + 2\sqrt{5} - \sqrt{3} + 7\sqrt{5} = 4\sqrt{3} + 9\sqrt{5}$$

$$5\sqrt{3} - 2\sqrt{12} = 5\sqrt{3} - 2(2\sqrt{3}) = 5\sqrt{3} - 4\sqrt{3} = \sqrt{3}$$

Addition – Simplify, then add like radicals.

1. $\sqrt{8} + \sqrt{2}$

2. $3\sqrt{5} + 2\sqrt{20}$

3. $4\sqrt{3} + \sqrt{75}$

4. $2\sqrt{7} + 5\sqrt{28}$

5. $3\sqrt{12} + 2\sqrt{27}$

6. $\sqrt{18} + 4\sqrt{2}$

7. $2\sqrt{45} + \sqrt{20}$

8. $5\sqrt{6} + 2\sqrt{24}$

Subtraction – Simplify, then subtract like radicals.

9. $5\sqrt{18} - 2\sqrt{8}$

10. $7\sqrt{12} - \sqrt{27}$

11. $6\sqrt{50} - 3\sqrt{2}$

12. $10\sqrt{3} - 2\sqrt{75}$

13. $4\sqrt{27} - 3\sqrt{12}$

14. $3\sqrt{40} - 2\sqrt{10}$

15. $2\sqrt{72} - 6\sqrt{2}$

16. $5\sqrt{32} - 4\sqrt{8}$

Add Radicals

Simplify each radical, then combine like radicals.

1. $3\sqrt{12} + 2\sqrt{27}$
2. $\sqrt{8} + \sqrt{18}$
3. $5\sqrt{32} + \sqrt{50}$
4. $7\sqrt{45} + 2\sqrt{20}$
5. $2\sqrt{75} + 4\sqrt{12}$
6. $4\sqrt{27} + 3\sqrt{12}$
7. $2\sqrt{18} + \sqrt{2}$
8. $\sqrt{6} + 2\sqrt{24}$
9. $3\sqrt{5} + 2\sqrt{20}$
10. $5\sqrt{28} + 3\sqrt{7}$

Subtract Radicals

Simplify each radical, then combine like radicals.

1. $5\sqrt{18} - 2\sqrt{8}$
2. $7\sqrt{50} - 3\sqrt{8}$
3. $\sqrt{45} - \sqrt{5}$
4. $4\sqrt{27} - \sqrt{12}$
5. $6\sqrt{32} - 2\sqrt{2}$
6. $3\sqrt{40} - \sqrt{10}$
7. $8\sqrt{18} - \sqrt{72}$
8. $2\sqrt{6} - \sqrt{24}$

Multiplying and Dividing Radicals

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{ab} \quad \text{and} \quad \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

Multiply or divide the values under the radical, then simplify.

Worked Examples

$\sqrt{3}\cdot\sqrt{5}=\sqrt{15}$

$\frac{\sqrt{18}}{\sqrt{3}}=\sqrt{6}$

$\sqrt{\frac{4}{9}}=\frac{\sqrt{4}}{\sqrt{9}}=\frac{2}{3}$

Simplify each product.

1. $\sqrt{6}\cdot\sqrt{3}$

2. $(2\sqrt{5})(3\sqrt{2})$

3. $(\sqrt{7})^2$

4. $(4\sqrt{2})(\sqrt{6})$

5. $\sqrt{12}\cdot\sqrt{15}$

6. $(3\sqrt{8})(2\sqrt{3})$

7. $(5\sqrt{3})(2\sqrt{12})$

8. $(2\sqrt{10})(3\sqrt{15})$

Simplify each quotient. Rationalize the denominator when necessary.

9. $\frac{5}{\sqrt{2}}$

10. $\frac{3}{\sqrt{5}}$

11. $\frac{4\sqrt{3}}{\sqrt{12}}$

12. $\frac{\sqrt{45}}{\sqrt{5}}$

13. $\frac{7}{2\sqrt{3}}$

14. $\frac{\sqrt{18}}{\sqrt{2}}$

15. $\frac{6\sqrt{5}}{\sqrt{15}}$

16. $\frac{\sqrt{32}}{\sqrt{2}}$

Multiplying and Dividing Radicals

Instructions: Simplify each expression. Write each answer in simplest surd form.

A. Multiply the Radicals

Rule

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

1. $\sqrt{3} \cdot \sqrt{12}$

2. $\sqrt{5} \cdot \sqrt{20}$

3. $\sqrt{2} \cdot \sqrt{18}$

4. $\sqrt{7} \cdot \sqrt{28}$

5. $\sqrt{10} \cdot \sqrt{50}$

6. $\sqrt{6} \cdot \sqrt{24}$

7. $\sqrt{8} \cdot \sqrt{27}$

8. $\sqrt{15} \cdot \sqrt{5}$

B. Divide the Radicals

Rule

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \qquad b > 0$$

9. $\dfrac{\sqrt{50}}{\sqrt{2}}$

10. $\dfrac{\sqrt{72}}{\sqrt{8}}$

11. $\dfrac{\sqrt{98}}{\sqrt{2}}$

12. $\dfrac{\sqrt{128}}{\sqrt{8}}$

13. $\dfrac{\sqrt{200}}{\sqrt{5}}$

14. $\dfrac{\sqrt{45}}{\sqrt{5}}$

15. $\dfrac{\sqrt{80}}{\sqrt{4}}$

16. $\dfrac{\sqrt{162}}{\sqrt{18}}$

Rationalizing Radicals

Rationalize	Remove the radical from the denominator of a fraction.

Rationalizing a Single Radical Denominator

For a denominator containing $c\sqrt{a}$, multiply the numerator and denominator by $\sqrt{a}$:

$$\frac{b}{c\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{b\sqrt{a}}{ca}.$$

This works because

$$\sqrt{a} \cdot \sqrt{a} = a.$$

Always simplify the resulting fraction and radical when possible.

Worked Examples

$$\frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

$$\frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

$$\frac{10}{2\sqrt{2}} = \frac{10\sqrt{2}}{4} = \frac{5\sqrt{2}}{2}$$

Rationalize each denominator. Simplify if possible.

1. $\frac{1}{\sqrt{2}}$

2. $\frac{5}{\sqrt{3}}$

3. $\frac{7}{\sqrt{5}}$

4. $\frac{3}{\sqrt{6}}$

5. $\frac{4}{2\sqrt{3}}$

6. $\frac{1}{2\sqrt{3}}$

7. $\frac{4}{3\sqrt{2}}$

8. $\frac{9}{2\sqrt{5}}$

9. $\frac{\sqrt{3}}{3\sqrt{5}}$

10. $\frac{2\sqrt{5}}{5\sqrt{3}}$

11. $\frac{\sqrt{7}}{3\sqrt{2}}$

12. $\frac{6\sqrt{2}}{5\sqrt{3}}$

13. $\frac{22}{4\sqrt{11}}$

14. $\frac{9\sqrt{2}}{3\sqrt{5}}$

15. $\frac{3\sqrt{2}}{5\sqrt{6}}$

16. $\frac{8\sqrt{3}}{3\sqrt{2}}$

Simple Denominators

Rationalize each denominator and simplify. Show all work.

1. $\frac{5}{\sqrt{3}}$

2. $\frac{2}{\sqrt{7}}$

3. $\frac{8}{\sqrt{5}}$

4. $\frac{7}{\sqrt{11}}$

5. $\frac{4}{\sqrt{15}}$

6. $\frac{9}{\sqrt{2}}$

7. $\frac{3}{2\sqrt{6}}$

8. $\frac{\sqrt{8}}{\sqrt{3}}$

9. $\frac{6}{\sqrt{12}}$

10. $\frac{11}{\sqrt{13}}$

11. $\frac{5}{\sqrt{18}}$

12. $\frac{7}{3\sqrt{2}}$

13. $\frac{12}{\sqrt{6}}$

14. $\frac{\sqrt{14}}{\sqrt{5}}$

15. $\frac{8}{\sqrt{20}}$

16. $\frac{10}{3\sqrt{5}}$

17. $\frac{\sqrt{15}}{2\sqrt{2}}$

18. $\frac{9}{2\sqrt{7}}$

Radical Conjugates

Conjugates	Two binomial expressions that have the same terms but opposite signs between the terms.

Conjugate Products

The expressions $a+b$ and $a-b$ are conjugates. Multiplying conjugates produces a difference of squares:

$$\boxed{(a+b)(a-b)=a^2-b^2}$$

For example, $3+\sqrt{2}$ and $3-\sqrt{2}$ are conjugates.

$$\begin{aligned}(3+\sqrt{2})(3-\sqrt{2}) &= 3^2-(\sqrt{2})^2\\ &= 9-2\\ &= 7\end{aligned}$$

The middle terms cancel, leaving an expression without a radical.

A. Write the conjugate of each expression.

1. $1+\sqrt{3}$ **2.** $\sqrt{2}-1$ **3.** $\sqrt{3}-\sqrt{2}$ **4.** $3+2\sqrt{2}$

B. Expand and simplify each conjugate product.

5. $(a+\sqrt{b})(a-\sqrt{b})$

6. $(\sqrt{a}+b)(\sqrt{a}-b)$

7. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

8. $(a+c\sqrt{b})(a-c\sqrt{b})$

C. Rationalize each denominator using its conjugate. Simplify fully.

9. $\dfrac{2}{\sqrt{2}+1}$ **10.** $\dfrac{5}{\sqrt{3}-2}$ **11.** $\dfrac{3}{1-\sqrt{5}}$ **12.** $\dfrac{6}{2+\sqrt{7}}$

13. $\dfrac{1}{\sqrt{10}-\sqrt{2}}$ **14.** $\dfrac{5+\sqrt{3}}{2-\sqrt{3}}$ **15.** $\dfrac{2\sqrt{5}}{1+\sqrt{5}}$ **16.** $\dfrac{4}{\sqrt{3}+\sqrt{2}}$

Binomial Denominators

Rationalize each denominator using the conjugate. Simplify fully and show all work.

17. $\dfrac{5}{3+\sqrt{2}}$

18. $\dfrac{4}{\sqrt{7}-2}$

19. $\dfrac{6}{5+\sqrt{3}}$

20. $\dfrac{3}{\sqrt{5}-1}$

21. $\dfrac{7}{\sqrt{2}+4}$

22. $\dfrac{8}{\sqrt{6}-3}$

23. $\dfrac{\sqrt{3}}{2+\sqrt{5}}$

24. $\dfrac{2\sqrt{2}}{\sqrt{3}-1}$

25. $\dfrac{5}{\sqrt{10}+3}$

26. $\dfrac{4}{2-\sqrt{8}}$

27. $\dfrac{7}{\sqrt{3}+2}$

28. $\dfrac{9}{5-\sqrt{2}}$

29. $\dfrac{4}{\sqrt{11}-3}$

30. $\dfrac{3\sqrt{2}}{4+\sqrt{5}}$

31. $\dfrac{6}{2-\sqrt{7}}$

32. $\dfrac{5}{\sqrt{13}+1}$

33. $\dfrac{2\sqrt{3}}{\sqrt{5}+2}$

34. $\dfrac{7}{3-\sqrt{6}}$

Binomial Numerators and Denominators

Rationalize each denominator using the conjugate. Simplify fully and show all work.

35. $\dfrac{\sqrt{3}+2}{\sqrt{3}-2}$

36. $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

37. $\dfrac{4+\sqrt{7}}{4-\sqrt{7}}$

38. $\dfrac{\sqrt{2}+5}{\sqrt{2}-5}$

39. $\dfrac{\sqrt{6}-3}{\sqrt{6}+3}$

40. $\dfrac{\sqrt{8}+1}{\sqrt{8}-1}$

41. $\dfrac{2-\sqrt{3}}{2+\sqrt{3}}$

42. $\dfrac{5+\sqrt{11}}{5-\sqrt{11}}$

43. $\dfrac{\sqrt{13}-4}{\sqrt{13}+4}$

44. $\dfrac{3+\sqrt{15}}{3-\sqrt{15}}$

45. $\dfrac{\sqrt{7}+3}{\sqrt{7}-3}$

46. $\dfrac{2+\sqrt{5}}{2-\sqrt{5}}$

47. $\dfrac{\sqrt{10}-1}{\sqrt{10}+1}$

48. $\dfrac{6+\sqrt{2}}{6-\sqrt{2}}$

49. $\dfrac{\sqrt{12}+5}{\sqrt{12}-5}$

50. $\dfrac{4-\sqrt{15}}{4+\sqrt{15}}$

51. $\dfrac{3+\sqrt{2}}{3-\sqrt{2}}$

52. $\dfrac{\sqrt{14}-2}{\sqrt{14}+2}$

Solving Equations

Equation	A mathematical statement showing that two expressions have the same value.

Steps for Solving an Equation

1. Expand brackets and collect like terms when needed.
2. Move variable terms to one side of the equation.
3. Use inverse operations to isolate the variable.
4. Perform the same operation on both sides to maintain balance.
5. Check the solution by substituting it into the original equation.

Whatever you do to one side, you must do to the other side.

Worked Examples

Example 1

$$3x + 7 = 22$$
$$3x + 7 - 7 = 22 - 7$$
$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Check: $3(5) + 7 = 22$

Example 2

$$\frac{x}{5} + 2 = -2$$
$$\frac{x}{5} + 2 - 2 = -2 - 2$$
$$\frac{x}{5} = -4$$
$$5\left(\frac{x}{5}\right) = 5(-4)$$
$$x = -20$$

Check: $\frac{-20}{5} + 2 = -2$

Solve each equation. Show all steps and check when possible.

1. $3x = 18$

2. $x - 4 = 9$

3. $\frac{x}{2} = 6$

4. $4x + 3 = 19$

5. $-2x + 5 = 1$

6. $\frac{x}{5} + 3 = 7$

7. $\frac{2x + 3}{5} = 4$

8. $2(x - 3) + 4 = 12$

9. $\frac{5(x + 2)}{3} = 10$

10. $3(x + 4) = 2x + 17$

11. $5x - 8 = 2x + 13$

12. $\frac{x - 2}{4} + 3 = 6$

One-Step Equations

Solve each equation. Show all work and check your solution.

1. $x + 7 = 15$

2. $y - 9 = 12$

3. $3z = 18$

4. $\frac{m}{5} = 6$

5. $a - 14 = -2$

6. $4b = 28$

7. $\frac{c}{7} = -3$

8. $d + 11 = -4$

9. $e - 6 = 25$

10. $-5f = 40$

11. $g + 13 = 2$

12. $h - 18 = -7$

13. $6j = -54$

14. $\frac{k}{8} = -5$

Two-Step Equations

Instructions: Solve each equation. Show each inverse operation and check your solution when possible.

Solve each two-step equation.

1. $2x + 5 = 13$

2. $3y - 7 = 11$

3. $4z + 6 = 18$

4. $5m - 9 = 16$

5. $\frac{a}{3} + 4 = 10$

6. $2b - 5 = -9$

7. $\frac{c}{4} - 7 = 2$

8. $6d + 3 = 15$

9. $-2e + 8 = 0$

10. $7f - 12 = 23$

11. $9g + 4 = 31$

12. $-3h - 5 = 16$

13. $\frac{j}{6} + 9 = 2$

14. $5k - 14 = -29$

Equations with Distribution

Solve each equation. Use the distributive property, show all work, and check your solution.

1. $3(x+4)=24$

2. $5(y-2)=35$

3. $-2(z+7)=10$

4. $4(m-6)=-8$

5. $2(3a-5)=22$

6. $-3(2b+1)=15$

7. $7(c-3)=2c+16$

8. $5(d+4)=3d+18$

9. $2(4e-3)=5e+9$

10. $-6(f-2)=3f+12$

11. $8(g-5)=3g+7$

12. $-4(h+6)=2h-20$

13. $3(2j-7)=j+11$

14. $5(k+1)=2(k+13)$

Fraction Coefficients

Instructions: Solve each equation. Show all steps and leave each answer in simplest form.

Reminder

To undo multiplication by a fraction, multiply both sides by its reciprocal.

$$\frac{3}{4}x = 6 \qquad \Longrightarrow \qquad x = 6\left(\frac{4}{3}\right)$$

Solve each equation.

1. $\frac{1}{2}x + 3 = 11$

2. $\frac{3}{4}y - 5 = 7$

3. $\frac{2}{3}z + 4 = 10$

4. $\frac{5}{6}m - 2 = 8$

5. $\frac{3}{5}a + \frac{1}{2} = 4$

6. $\frac{7}{8}b - 1 = -\frac{1}{4}$

7. $\frac{2}{5}c - \frac{3}{2} = \frac{1}{10}$

8. $-\frac{3}{4}d + \frac{5}{6} = 2$

9. $\frac{5}{3}e - \frac{1}{3} = 4$

10. $-\frac{2}{7}f + 3 = \frac{9}{7}$

11. $\frac{2}{9}g + 5 = \frac{7}{3}$

12. $-\frac{4}{5}h - \frac{1}{2} = \frac{3}{10}$

Multi-Step Equations

Instructions: Solve each equation. Expand brackets, collect like terms, and show all steps. Check each solution when possible.

Solve each multi-step equation.

1. $3x + 7 = 2x - 5$

2. $5y - 9 = 2y + 12$

3. $4z + 6 = 2(z + 9)$

4. $7m - 3 = 5(m + 1)$

5. $2(2a - 3) + 5 = 3a + 1$

6. $6b - 2(3b - 4) = 18$

7. $8c + 5 = 3(c - 7) + 2c$

8. $9d - 4 = 2(d + 6) + 3d$

9. $-2e + 11 = 5 - (e - 3)$

10. $10f - 7 = 4(2f + 1) - f$

11. $3g - 2(g + 5) = 7 - 4g$

12. $5(h - 3) + 2h = 4(h + 6) - 7$

Challenge Equations

Solve each equation. Show all algebraic steps and check your solution.

1. $\frac{3}{2}(x-4)+\frac{1}{3}x=7$

2. $4(2y-3)-5(y+1)=2y+9$

3. $\frac{z-5}{4}+\frac{z+1}{6}=3$

4. $3(2m+1)-2(5-m)=4m-7$

5. $\frac{2}{5}a-\frac{3}{10}=\frac{1}{2}(a-1)$

6. $7(b-2)-3(2b+5)=-(b-1)$

7. $\frac{c}{3}-\frac{c-6}{5}=2$

8. $2(d+4)-\frac{3}{2}(d-2)=11$

9. $\frac{e+2}{5}+\frac{e-1}{2}=\frac{7}{2}$

10. $3(f-4)-\frac{1}{2}(2f+6)=\frac{5}{2}$

Linear Equations

Linear Equation	An equation in which each variable has an exponent of 1, with no squares, roots, or products of variables. Its graph is a straight line.

Steps for Solving Linear Equations

1. Simplify both sides by expanding brackets and combining like terms.
2. Move all variable terms to one side.
3. Move all constant terms to the opposite side.
4. Use inverse operations to isolate the variable.
5. Examine the final statement for a special solution case.

Types of Solutions for Linear Equations

A linear equation can have **one solution**, **no solution**, or **infinitely many solutions**. Simplify both sides and examine the final statement.

Example 1 – One Solution

Solve $3x + 5 = 2x + 8$.

$$\begin{aligned} 3x + 5 &= 2x + 8 \\ 3x - 2x + 5 &= 2x - 2x + 8 \\ x + 5 &= 8 \\ x &= 3 \end{aligned}$$

ONE SOLUTION

The variable remains in the final equation.

$$\boxed{x = 3}$$

Only one value of x makes the equation true.

Example 2 – No Solution

Solve $4x + 2 = 4x - 5$.

$$\begin{aligned} 4x + 2 &= 4x - 5 \\ 4x - 4x + 2 &= 4x - 4x - 5 \\ 2 &= -5 \end{aligned}$$

NO SOLUTION

The variables cancel and the final statement is false.

$$\boxed{2 \neq -5}$$

No value of x makes the equation true.

Example 3 – Infinitely Many Solutions

Solve $5x - 3 = 5x - 3$.

$$\begin{aligned} 5x - 3 &= 5x - 3 \\ 5x - 5x - 3 &= 5x - 5x - 3 \\ -3 &= -3 \end{aligned}$$

INFINITELY MANY SOLUTIONS

The variables cancel and the final statement is true.

$$\boxed{-3 = -3}$$

Every real value of x makes the equation true.

Number of Solutions – Practice

Determine whether each equation has one solution, no solution, or infinitely many solutions. Show enough work to justify your answer.

1. $2x + 7 = 3x - 1$

2. $4(x - 2) = 4x - 8$

3. $5x + 3 = 5x - 9$

4. $3(x + 2) = 2x + (x + 7)$

5. $7 - 2x = 3x + 12$

6. $-3(2x - 1) + 4 = -6x + 7$

7. $8x - (2x + 5) = 6x - 5$

8. $9(x - 1) = 9x + 2$

9. $\frac{3}{2}x - 4 = \frac{1}{2}x + 6$

10. $2(x - 3) + 5 = x + 1$

Radical Equations

Radical Equation	An equation in which the variable appears inside a radical. Solving usually requires isolating the radical and raising both sides to the appropriate power.

Steps for Solving a Radical Equation

Step 1 Isolate the radical → **Step 2** Raise both sides to a power → **Step 3** Simplify the resulting equation → **Step 4** Solve the equation → **Step 5** Check in the original equation

Worked Examples

Example 1: One Valid Solution

Solve:

$$\sqrt{x+3} = 5$$

Square both sides:

$$x + 3 = 25$$

$$x = 22$$

Check in the original equation:

$$\sqrt{22+3} = \sqrt{25} = 5$$

$$\boxed{x = 22}$$

Example 2: Extraneous Solution

Solve:

$$\sqrt{x+6} = x$$

Square both sides:

$$x + 6 = x^2$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

Candidates: $x = 3$ or $x = -2$.

$$\sqrt{3+6} = 3 \qquad \checkmark$$

$$\sqrt{-2+6} \neq -2 \qquad \times$$

$$\boxed{x = 3}$$

Extraneous solution: A value produced during the algebraic process that does not satisfy the original equation. Raising both sides to an even power can introduce extraneous solutions, so every candidate must be checked.

Quick Practice

Solve each radical equation. Check every solution in the original equation.

1. $\sqrt{x} = 7$

2. $\sqrt{x+5} = 6$

3. $\sqrt{2x-1} = 5$

4. $\sqrt{x-4} + 2 = 7$

5. $\sqrt{x+6} = x$

6. $\sqrt{2x+3} = x$

Solving Radical Equations Practice

Instructions: Solve each radical equation. Show all steps and check every possible solution in the original equation for extraneous solutions.

Remember

Isolate the radical, raise both sides to the required power, solve the resulting equation, and check all possible solutions in the original equation.

Solve each radical equation.

1. $\sqrt{x} = 5$

2. $\sqrt{x+3} = 4$

3. $\sqrt{x-7} = 6$

4. $\sqrt{2x} = 8$

5. $\sqrt{x+1} = 7$

6. $\sqrt{3x-2} = 4$

7. $\sqrt{x-5} + 1 = 4$

8. $\sqrt{x+6} - 2 = 5$

9. $\sqrt{2x+1} = 9$

10. $\sqrt{x-4} + 3 = 8$

11. $\sqrt{x+2} = x - 2$

12. $\sqrt{2x-1} = x - 1$

13. $\sqrt{x+5} + 2 = x$

14. $\sqrt{x-1} + \sqrt{x-9} = 6$

Solving Inequalities

Inequality	A mathematical statement that compares two expressions using $<$, $>$, $\leq$, or $\geq$.

Key Rule

Solve an inequality using the same inverse operations used to solve an equation.

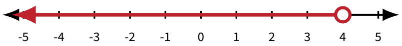
Reverse the inequality symbol whenever both sides are multiplied or divided by a negative number.

$$-3x \leq 9 \quad \Longrightarrow \quad x \geq -3$$

Worked Examples

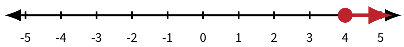
Example 1: Addition

$$x + 3 < 7$$
$$x < 4$$



Example 2: Two-Step Inequality

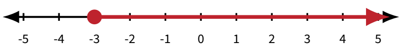
$$2x - 5 \geq 3$$
$$2x \geq 8$$
$$x \geq 4$$



Example 3: Dividing by a Negative

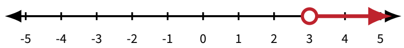
$$-3x \leq 9$$
$$x \geq -3$$

Divide by -3 and reverse the symbol.



Example 4: Two-Step Inequality

$$4x + 7 > 19$$
$$4x > 12$$
$$x > 3$$



Graphing Inequalities

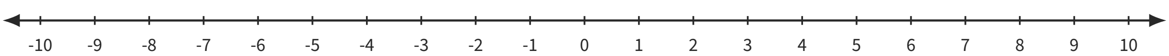
○ **Open circle:** Use an open circle for $<$ or $>$ because the endpoint is **not included** in the solution.

● **Closed circle:** Use a closed circle for $\leq$ or $\geq$ because the endpoint is **included** in the solution.

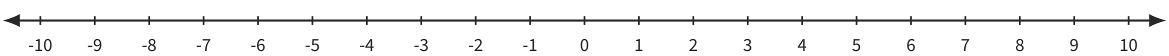
Quick Practice

Solve each inequality and graph the solution on the number line.

1. $3x + 5 \leq 14$



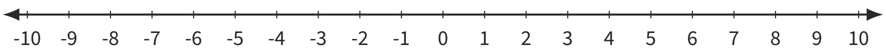
2. $-4x + 2 > 10$



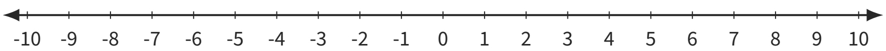
Solving Inequalities Practice

Instructions: Solve each inequality. Show all algebraic steps, then graph the solution on the number line.

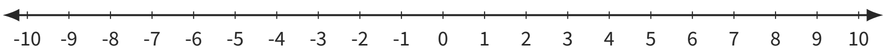
1. $x - 5 > -2$



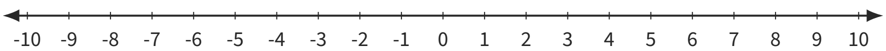
2. $3x + 4 \leq 10$



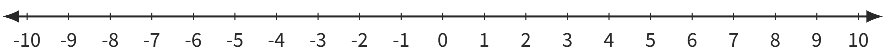
3. $-2x > 8$



4. $5x - 7 < 18$



5. $7 - 2x \geq -3$



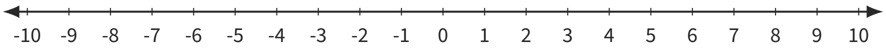
Solving Inequalities Practice – Continued

Instructions: Solve each inequality. Show all algebraic steps, then graph the solution on the number line.

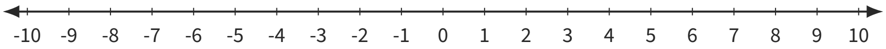
Remember

Reverse the inequality symbol whenever both sides are multiplied or divided by a negative number.

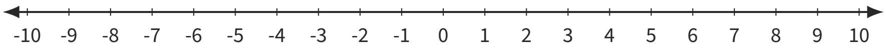
6. $\frac{1}{2}x - 3 < 5$



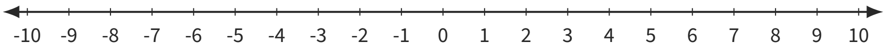
7. $4x + 2 \leq -6$



8. $-5x + 9 > 4$



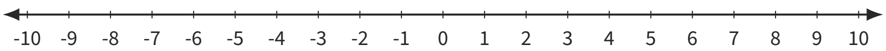
9. $6x - 8 \geq 10$



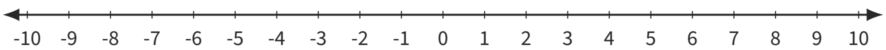
Solving Inequalities Practice – Continued

Instructions: Solve each inequality. Show all algebraic steps, then graph the solution on the number line.

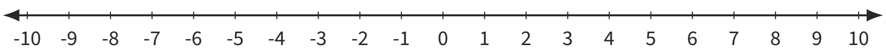
10. $2(x-3) < 8$



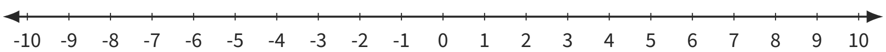
11. $3(x+2) \geq 9$



12. $-4(x-1) < 12$



13. $5-2(x+7) \leq -9$



Compound Inequalities

Compound Inequality	Two inequalities joined by the word **and** or the word **or**.

AND The solution must satisfy **both inequalities**. Keep and graph only the **overlapping region**.

OR The solution may satisfy **either inequality**. Keep and graph **both solution regions**.

Worked Examples

Example 1: AND

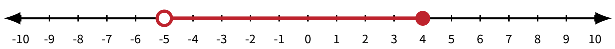
$$-2 < x + 3 \leq 7$$

Subtract 3 from all three parts:

$$-5 < x \leq 4$$

Interval notation:

$$(-5, 4]$$



Example 2: OR

$$2x - 1 < 5 \quad \text{or} \quad 2x - 1 \geq 9$$

$$x < 3 \quad \text{or} \quad x \geq 5$$

Interval notation:

$$(-\infty, 3) \cup [5, \infty)$$

Example 3: AND

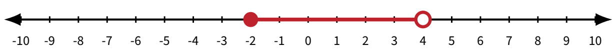
$$-6 \leq 3x < 12$$

Divide all three parts by 3:

$$-2 \leq x < 4$$

Interval notation:

$$[-2, 4)$$



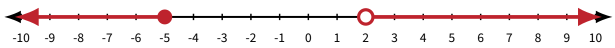
Example 4: OR

$$x + 4 \leq -1 \quad \text{or} \quad x + 4 > 6$$

$$x \leq -5 \quad \text{or} \quad x > 2$$

Interval notation:

$$(-\infty, -5] \cup (2, \infty)$$

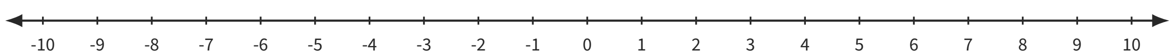


Quick Practice

Solve each compound inequality. Write the solution in interval notation and graph it on the number line.

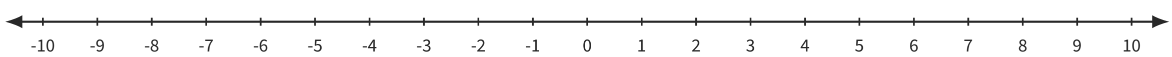
1. $-1 < 2x + 3 \leq 9$

Solution: ____________________ **Interval:** ____________________



2. $3x + 2 < -4$ or $3x + 2 \geq 11$

Solution: ____________________ **Interval:** ____________________



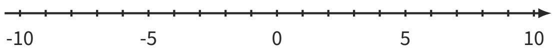
Compound Inequalities Practice

Instructions: Solve each compound inequality when needed. Show your algebraic steps, then graph the solution on the number line.

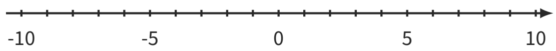
Remember

For an **AND** inequality, graph the overlap of the two conditions. For an **OR** inequality, graph all values that satisfy either condition. Reverse the inequality symbol whenever you multiply or divide by a negative number.

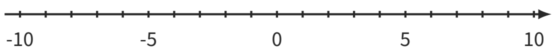
1. $-3 < x \leq 5$



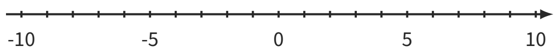
2. $x > 2$ or $x < -1$



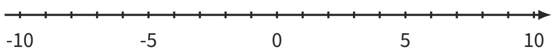
3. $0 \leq x < 4$



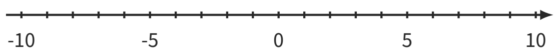
4. $x \geq -2$ and $x \leq 3$



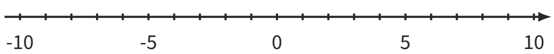
5. $2 < x + 1 \leq 6$



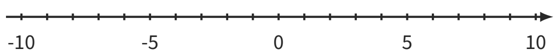
6. $-5 \leq 3x \leq 9$



7. $x - 4 > -3$ and $x + 2 \leq 7$



8. $\frac{x}{2} < 5$ or $x - 1 > 8$



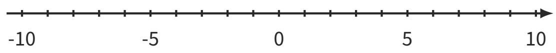
Compound Inequalities Practice – Continued

Instructions: Solve each compound inequality. Show your algebraic steps, then graph the solution on the number line.

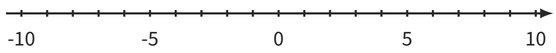
Remember

For an **AND** inequality, graph the overlap. For an **OR** inequality, graph both solution regions.

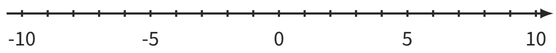
9. $-4 < x - 2 \leq 3$



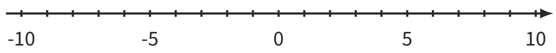
10. $1 \leq 2x + 5 < 11$



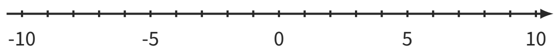
11. $3x - 1 > 8$ or $x + 4 < -2$



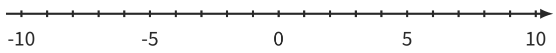
12. $-6 \leq \dfrac{x}{2} \leq 4$



13. $2x + 1 \geq -5$ and $x - 3 < 4$



14. $\dfrac{x - 1}{3} > 2$ or $2x + 5 \leq -1$



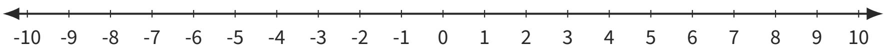
Compound Inequalities Practice

Instructions: Solve each compound inequality. Show all algebraic steps, then graph the solution on the number line.

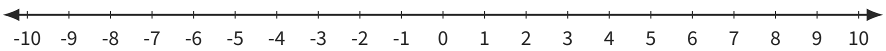
Remember

Apply the same operation to all three parts of the inequality. Reverse both inequality symbols when multiplying or dividing by a negative number.

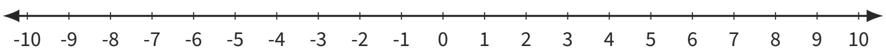
1. $-3 < x \leq 5$



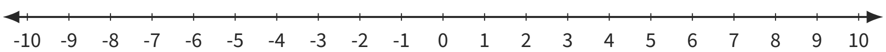
2. $-6 \leq x < 2$



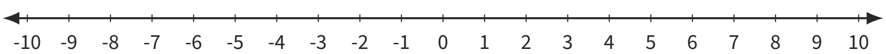
3. $1 < x + 4 \leq 9$



4. $-8 \leq x - 3 < 4$



5. $-10 < 2x \leq 6$



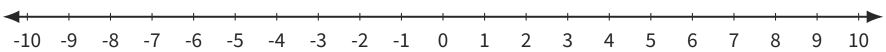
Compound Inequalities Practice – Continued

Instructions: Solve each compound inequality when needed. Show all algebraic steps, then graph the solution on the number line.

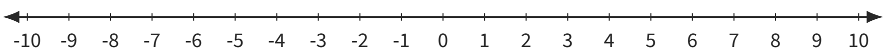
Remember

Apply the same operation to all three parts of a compound inequality.
For an **OR** inequality, graph both solution regions. Reverse the inequality symbol whenever multiplying or dividing by a negative number.

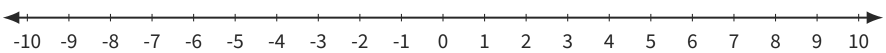
6. $-12 \leq 3x < 15$



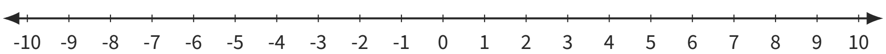
7. $-5 < \dfrac{x}{2} \leq 4$



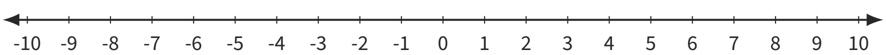
8. $-3 \leq \dfrac{x+1}{2} < 5$



9. $x < -2$ or $x > 4$



10. $x \leq -5$ or $x \geq 3$

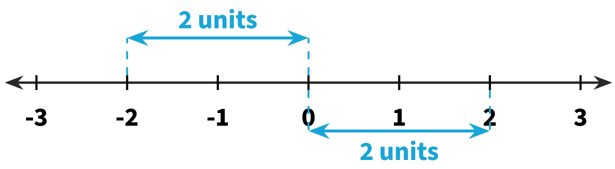


Absolute Value

Absolute Value	The distance a number is from zero

Absolute value describes the size of a number without considering its sign. It represents the distance between a number and 0 on a number line. Vertical bars are used to show absolute value:

$$|-4| = 4.$$



Absolute value measures distance from zero. Since distance cannot be negative, an absolute value is always non-negative.

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

Worked Examples

Apply the order of operations before evaluating the absolute value.

a $|-6 \times (-4)|$

$$\begin{aligned} |-6 \times (-4)| &= |24| \\ &= 24 \end{aligned}$$

b $|4 - 3 \times 7|$

$$\begin{aligned} |4 - 3 \times 7| &= |4 - 21| \\ &= |-17| \\ &= 17 \end{aligned}$$

c Solve $|x - 3| = 8$

$x - 3 = 8$ or $x - 3 = -8$

$x = 11$ or $x = -5$

Evaluate

1. $|7|$

2. $|-3|$

3. $|-12 + 5|$

4. $|4 - 9|$

Simplify

1. $2\,|3 - 7|$

2. $|5| + |-8|$

3. $4 - |2 - 6|$

4. $|3| - |-2|$

Solve

1. $|x| = 5$

2. $|x - 4| = 6$

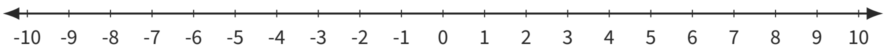
3. $|2x + 1| = 7$

4. $|x| = -3$

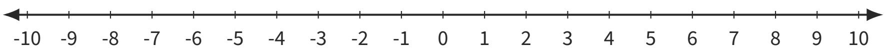
Absolute Value Inequalities Practice

Instructions: Rewrite each absolute value inequality as a compound inequality. Solve when needed, then graph the solution on the number line.

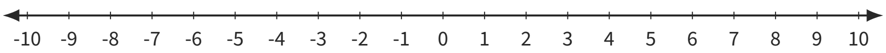
1. $|x| < 4$



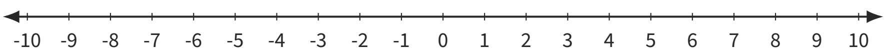
2. $|x| \geq 6$



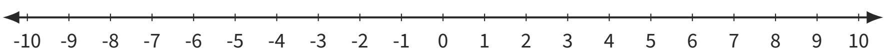
3. $|x - 3| \leq 5$



4. $|x + 2| > 7$



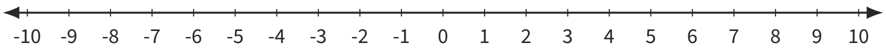
5. $|x - 5| < 3$



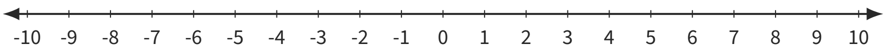
Absolute Value Inequalities Practice – Continued

Instructions: Rewrite each absolute value inequality as a compound inequality. Solve, then graph the solution on the number line.

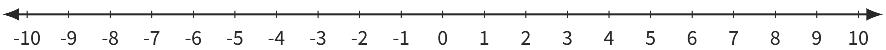
6. $|x+1| \geq 4$



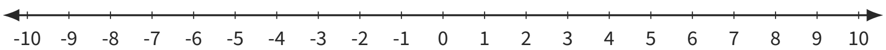
7. $|2x| < 8$



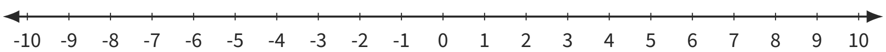
8. $|3x| \leq 12$



9. $|2x-1| < 7$



10. $|3x+6| \geq 9$



Oral Equations – Part A: Linear Equations

Instructions: Translate each statement into an algebraic equation, then solve. Define the variable and show all algebraic steps.

Translate each statement into an equation, then solve.

1. Seven more than half a number is 15.

2. Twice the sum of a number and 4 is 30.

3. Three times the difference of a number and 5 is 21.

4. One-third of a number increased by 8 is 17.

5. Four less than twice a number equals three times the number plus 2.

6. The sum of a number and 9 is twice the number minus 4.

7. The difference between three times a number and 7 is 26.

Oral Equations – Part B: Multi-Step Equations

Instructions: Translate each statement into an algebraic equation, then solve. Define the variable, use brackets where needed, and show all algebraic steps.

Translate each statement into an equation, then solve.

8. Three times the sum of a number and 4 equals twice the number plus 17.

9. Five times the difference between a number and 2 equals three times the number plus 10.

10. One-half of the sum of a number and 8 equals 12.

11. Three-fourths of a number minus 5 equals 10.

12. Twice the difference between a number and 7 equals four times 3.

13. Seven more than one-third of a number equals twice the number minus 11.

14. The sum of twice a number and 5 equals the difference between five times the number and 16.

15. Four times the sum of a number and 2 equals three times the sum of the number and 8.

Oral Equations – Parts C and D

Instructions: Translate each statement into an algebraic equation, then solve. Define the variable, show all algebraic steps, and check each solution in the original equation.

Part C: Absolute Value Equations

16. The distance between a number and 3 is 8.

17. Twice the distance between a number and 4 is 18.

18. The distance between twice a number and 5 is 9.

19. Three times the distance between a number and 2 is 21.

Part D: Radical Equations

20. The square root of a number is 9.

21. Five more than the square root of a number is 13.

22. Twice the square root of a number equals 14.

23. Three less than the square root of a number equals 5.

Oral Equations – Part E: Challenge

Instructions: Translate each statement into an algebraic equation, then solve. Define the variable, show all algebraic steps, and check each solution in the original equation.

Translate each statement into an equation, then solve.

24. The square root of the quantity $x + 4$ is 7.

25. Three more than the square root of a number is 11.

26. One more than twice the square root of a number equals 13.

27. The square root of a number minus the square root of 16 equals 5.

28. The absolute value of twice a number minus 7 equals 9.

29. The sum of a number and its reciprocal is $\frac{5}{2}$.

30. One more than the reciprocal of a number equals one-half.

31. The square of a number decreased by 5 equals 44.

Real-World Modeling

Worked Examples

Example 1: Writing and Solving an Equation

A taxi charges a fixed fee of $\$5.25$ plus $\$2.40$ per kilometre. The total fare is $\$22.05$. Find the distance travelled.

Define the variable
Let k represent the number of kilometres travelled.

Write the equation

$$5.25 + 2.40k = 22.05$$

Solve

$$\begin{aligned} 5.25 + 2.40k &= 22.05 \\ 2.40k &= 16.80 \\ k &= 7 \end{aligned}$$

Interpret the solution
The taxi travelled

$$\boxed{7\text{ km}}$$

Example 2: Writing and Solving an Inequality

A recreation centre charges $\$18$ for registration and $\$7$ per class. Maya can spend at most $\$60$. Find the greatest number of classes she can attend.

Define the variable
Let c represent the number of classes Maya attends.

Write the inequality
Since Maya can spend **at most** $\$60$,

$$18 + 7c \leq 60$$

Solve

$$\begin{aligned} 18 + 7c &\leq 60 \\ 7c &\leq 42 \\ c &\leq 6 \end{aligned}$$

Interpret the solution
Maya can attend at most

$$\boxed{6\text{ classes}}$$

Example 3: Comparing Two Cost Models

Company A charges $\$30$ plus $\$0.45$ per kilometre. Company B charges $\$48$ plus $\$0.25$ per kilometre. Find the distance at which the two plans cost the same.

Define the variable
Let d represent the number of kilometres travelled.

Write the equation

$$30 + 0.45d = 48 + 0.25d$$

Solve

$$\begin{aligned} 30 + 0.45d &= 48 + 0.25d \\ 0.20d &= 18 \\ d &= 90 \end{aligned}$$

Interpret the solution
The two plans cost the same after

$$\boxed{90 \text{ km}}$$

At 90 km, each plan costs

$$30 + 0.45(90) = 48 + 0.25(90) = \$70.50.$$

Example 4: Modeling a Profit Goal

A school club sells tickets for $\$14$ each. The event costs $\$320$ to run. The club wants to earn a profit of at least $\$800$. Find the minimum number of tickets that must be sold.

Define the variable
Let t represent the number of tickets sold.

Write the inequality

$$\text{profit} = \text{revenue} - \text{cost}$$

Therefore,

$$14t - 320 \geq 800$$

Solve

$$\begin{aligned} 14t - 320 &\geq 800 \\ 14t &\geq 1120 \\ t &\geq 80 \end{aligned}$$

Interpret the solution
The school club must sell at least

$$\boxed{80 \text{ tickets}}$$

Real-World Equations and Inequalities

Instructions: Define a variable, write an equation or inequality, solve it, and interpret the answer in context.

Model and solve each real-world problem.

1. You earn $\$15$ per hour babysitting. How many hours must you work to earn $\$180$?

2. A gym charges a $\$40$ sign-up fee plus $\$25$ per month. How many months can you afford if you have $\$165$?

3. A rental car company charges $\$50$ per day plus $\$0.20$ per kilometre. You pay $\$130$ after driving 150 km. For how many days did you rent the car?

4. A streaming service costs $\$12$ per month. You want to spend no more than $\$100$. For how many complete months can you subscribe?

5. A store sells T-shirts for $\$8$ each. You have $\$60$. What is the greatest number of T-shirts you can buy?

6. A school club has already raised $\$85$ and earns $\$18$ for each item it sells. How many items must the club sell to raise at least $\$265$?

Real-World Equations and Inequalities – Continued

Instructions: Define a variable, write an equation or inequality, solve it, and interpret the answer in context.

Interpreting Whole-Number Answers

When the variable represents people, trips, items, or complete hours, interpret the solution carefully.

- Round **down** when finding the greatest amount that can be afforded.
- Round **up** when finding the minimum amount needed to reach a goal.

Model and solve each real-world problem.

7. You have a $\$250$ gift card. You spend $\$32.75$ each time you go shopping. What is the greatest number of full shopping trips you can make?

8. A phone plan includes 5 GB of data and charges $\$10$ for each additional gigabyte. You used 8.5 GB and paid $\$75$. What is the monthly base fee?

9. A plumber charges $\$80$ for the first hour and $\$50$ for each additional hour. If the total bill is $\$230$, how many hours did the plumber work?

10. A school sells pizza slices for $\$3.50$ each. How many slices must be sold to raise at least $\$500$?

11. A bus rental costs $\$500$ plus $\$12$ per student. If the group has at most $\$980$, what is the greatest number of students who can go?

12. A taxi charges a $\$6$ starting fee plus $\$3$ per kilometre. You have no more than $\$42$. What is the greatest number of kilometres you can travel?

Student Notes

Student Notes

Student Notes

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