MATHEMATICAL INVESTIGATION

The 100 Lockers Problem

PATTERNS

FACTORS

PROOF

Name: _____ Section: _____

The situation
A school has 100 closed lockers numbered 1 to 100. One hundred students walk past the lockers, one at a time. Each student changes the state of certain lockers: an open locker is closed, and a closed locker is opened.

Student	Action
1	Changes every locker.
2	Changes every 2nd locker: 2, 4, 6, 8, . . .
3	Changes every 3rd locker: 3, 6, 9, 12, . . .
n	Changes every nth locker: $n, 2n, 3n, \ldots$

The question

After all 100 students have taken their turn, which lockers are open? How many lockers are open? Explain why your answer must always be true.

Your initial prediction – do not calculate everything yet.

Part A • Test a smaller case

Work with only lockers 1 to 20 and students 1 to 20. Use **O** for open and **C** for closed. Record the state after each of the first ten students.

After	Locker number																			
student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																				
2																				
3																				
4																				
5																				
6																				
7																				
8																				
9																				
10																				

Part A • Complete the small case

Students 11 to 20 each change only the lockers that are multiples of their own number. Complete the process, then record the final open lockers.

1	2	3	4	5	6	7	8	9	10
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

Open lockers from 1 to 20:

Part B • What controls each locker?

A locker changes only when a student's number divides the locker number exactly. For example, locker 12 is changed by students 1, 2, 3, 4, 6, and 12.

Locker	Students who change it	Number of changes	Final state
6			
8			
9			
12			
16			
18			
20			
25			
36			

1. What determines whether a locker finishes open or closed?

2. What do the open lockers have in common?

Part C • Generalize to 100 lockers

Use your work from the smaller case and factor table. You should not need to simulate all 100 students.

1. List every locker that you believe will remain open.

2. How many lockers remain open? Explain how you know your list is complete.

3. Why do most factors occur in pairs?

4. When does a factor pair contain the same number twice?

Part D • Build the argument

Claim
State exactly which lockers remain open.

Evidence
Use examples involving factors and the number of times a locker changes.

Reasoning
Explain why your pattern works for every locker from 1 to 100.

Challenge • Extend the pattern

Suppose there are N lockers and N students following the same rule.

a) Write a rule for the number of lockers that remain open.

b) If exactly 15 lockers remain open, what is the smallest possible value of N? Justify your answer.