LEVERAGING STUDENT STRATEGIES THROUGH NUMBER TALKS

CHAD WILLIAMS
@CHADILLAC60
CHAD@BEYONDTHEALGORITHM.CA

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NUMBER TALKS

What is a Number Talk?

A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. A Number Talk is a powerful tool for helping students develop computational fluency because the expectation is that they will use number relationships and the structures of numbers to add, subtract, multiply and divide

Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**. In fact, teachers need to spend only 5 to 15 minutes on Number Talks. Number Talks are most effective when done everyday.

What is the Goal of Number Talks?

The primary goal of Number Talks is computational fluency.

Children develop computational fluency while thinking and reasoning like mathematicians. Children are asked to make connections and look for relationships and thus are engaged in "doing mathematics." When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing

In order for children to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures. Students need to understand that:

- Numbers are composed of smaller numbers.
- Numbers can be taken apart and combined with other numbers to make new numbers.
- What we know about one number can help us figure out other numbers.
- What we know about parts of smaller numbers can help us with parts of larger numbers.
- Numbers are organized into groups of tens and ones (and hundreds, tens and ones and so forth.)
- What we know about numbers to 10 helps us with numbers to 100 and beyond

What is Computational Fluency?

The National Council for Teachers of Mathematics states that "Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships."

Principal and Standards for School Mathematics, NCTM, Reston, VA 2000, p.152

A Number Talk is a powerful tool for helping students develop computational fluency because the expectation is that they will use number relationships and the structures of numbers to add, subtract, multiply and divide.

What is the format for Number Talks?

Number Talks are a large or small group meeting where the teacher poses intentionally selected problems for students to solve**.** They are short, ongoing conversations where children are encouraged to add, subtract, multiply and divide in ways that are meaningful to them, rather than following procedures that are not.

All Number Talks follow a basic six-step format. The format is the same, but the problems and models used will differ for each number talk.

1. Teacher presents the problem.
Problems are presented in many different ways: as dot cards, ten frames, sticks of cubes, models shown on the overhead, a word problem or a written problem such as
5 – 2 or 223 + 129 or 45% of 60.

2. Students figure out the answer.
Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal is unobtrusive- a message to the teacher, not the other students.

3. Students share their answers.
Four or five students volunteer to share their answers and the teacher records them on the board.

4. Students share their thinking.
Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.

5. The class agrees on the "real" answer for the problem.
The answer that the class together determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."

6. The steps are repeated for additional problems.

There are several elements that must be in place to ensure students get the most from their Number Talk experiences.

- **A safe environment**
- **Problems of various levels of difficulty that can be solved in a variety of ways**
- **Concrete models**
- **Opportunities to think first and then check**
- **Interaction**
- **Self-correction**

What is the Teacher's Role during Number Talks?

Teachers do not teach specific strategies to the children during Number Talks because it is important that the children think for themselves and use the mathematics they determine is most applicable to the problem at hand.

The teacher's job is to carefully select and present the types of problems that make the particular mathematics and number relationships evident to the students. Through focused, frequent and ongoing experiences, the children learn the mathematics necessary for computational fluency.

The teacher can meet the various levels of thinking that students have reached by providing problems of varying degrees of difficulty. However, for all children to gain from Number Talks, everyone should have access to the problems presented. That is, the children should be able to work on the problem in some way. Teachers can make sure children have access to the problems in three ways: 1) by allowing them to solve problems in their own ways; 2) by presenting problems at varying levels of difficulty ensuring that every child is able to solve at least some of the problems correctly and 3) by providing models for support.

The concrete models should help children develop more and more efficient strategies as they learn to take numbers apart and as they recognize particular relationships among the numbers. The models need to be aids to thinking, not tools for getting answers or to demonstrate memorized procedures. Children should become less and less dependent on the use of the models for any particular concept, ultimately not needing the model at all.

During a Number Talk, the interaction between teacher and students should be like a conversation rather than a report. When the children are explaining their thinking, the teacher must be genuinely interested in what the children are saying. The teacher naturally interacts with the children, helping them to clarify and communicate the process they have used. Teachers help students clarify their thinking in several ways: by asking questions, by describing what the child did, and by writing down the process.

The teacher

- Provides a safe environment where each child's thinking is valued.
- Selects groups or strings of problems that allow access to all children.
- Selects problems that intentionally highlight mathematical concepts.
- Values everyone's thinking, focusing on how children get their answers.
- Provides adequate wait time.
- Shifts the focus from, "See what I see," to "What do YOU (the child) see?"
- Records, clarifies, restates.
- Realizes that it if the children don't get it, then it is the teacher's responsibility to figure out their misconceptions or lack of proficiency and to begin instruction at that point.

The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?

Progression to Addition & Subtraction

Addition & Subtraction

Date you start:

Addition Strategies

Subtraction Strategies

COUNTING PRINCIPLES

"Many of the mathematical concepts that students learn in the first few years of school are closely tied to counting. The variety and accuracy of children's counting strategies and the level of their skill development in counting are valuable indicators of their growth in mathematical understanding in the primary years".

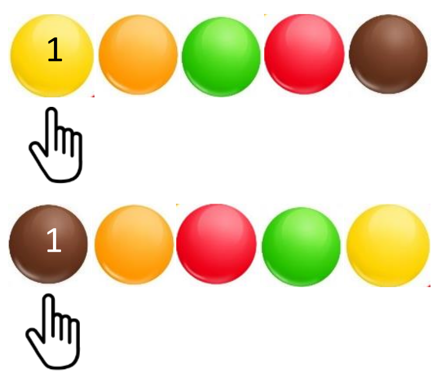
Guide to Effective Instruction: Number Sense and Numeration, 2016.

The following are basic concepts embedded in the early understanding of counting. Children will not learn these concepts in a linear fashion but will move back and forth through these concepts learning parts of some before moving on to another concept.

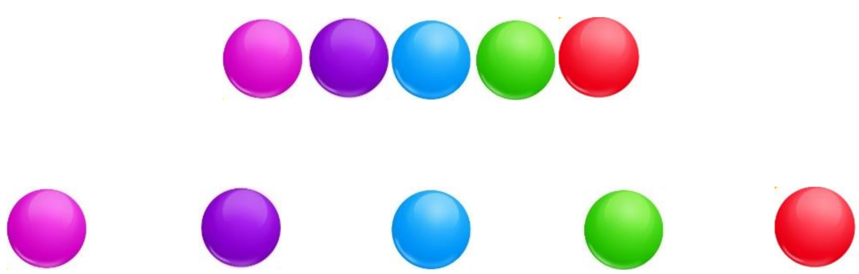
Stable Order: The counting sequence stays consistent or "stable".

1 2 3 4 5 6 7 8 9 etc…

Order Irrelevance: Counting objects can begin with any object in the set and the total will be the same. The order in which items are counted is irrelevant.



Conservation: The count for a set group stays the same no matter whether the distance between the objects are spread out or close together.



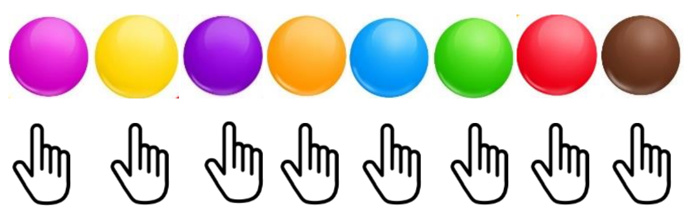
Counting Principles

Subitizing: The ability to see small amounts of objects without counting.

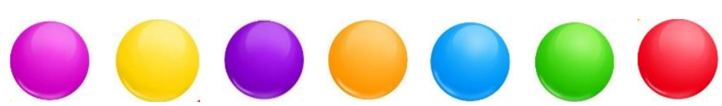
- Perceptual Subitizing: perceive objects without counting.
- Conceptual Subitizing: subitize two or more amounts perceptually and combine the amounts automatically.

One-to-One Correspondence: Each object being counted in a group can be counted once and only once.

1 2 3 4 5 6 7 8



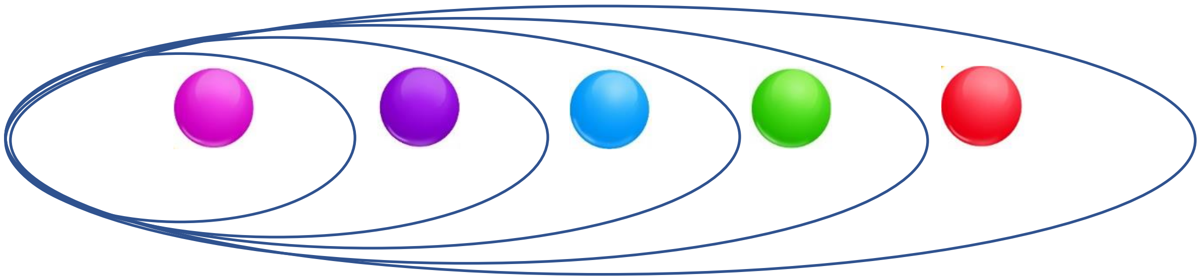
Cardinality: The last count of a group of objects represents the total number of objects in the group.



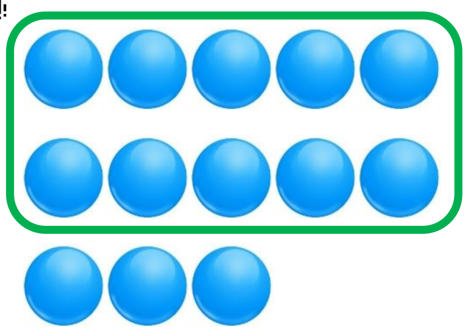
1 2 3 4 5 6 7 … 7 in all

Counting Principles

Hierarchical Inclusion: The idea that smaller numbers are nested in bigger numbers. The notion that each number includes those that came before.



Unitizing: The idea that objects are grouped into certain units. You can count a large group of items by decomposing the group into smaller, equal groups of items and then count those.



Tens	Ones
3	2

Number Patterns

Name:______________________________________

Patterns Check In

1. Fill in the missing numbers. Write the pattern rule.

a) 15, 20, 25, ____, ____, ____

.____.

b) 40, 50, 60,____, ____, ____

.____.

2.

a) Write the pattern rule.

.____.

b) Draw a pattern that has the same starting point but increases a different way.

Name:_______________________________________

3. Fill in the missing numbers. Write the pattern rule.

a) 35, 30, 25, 20, ___, ___, ___

.____.

b) 83, 80, 77, ___, ___, ___, ___

.____.

4. Janie made a decreasing pattern out of beads.
   She continues the pattern.

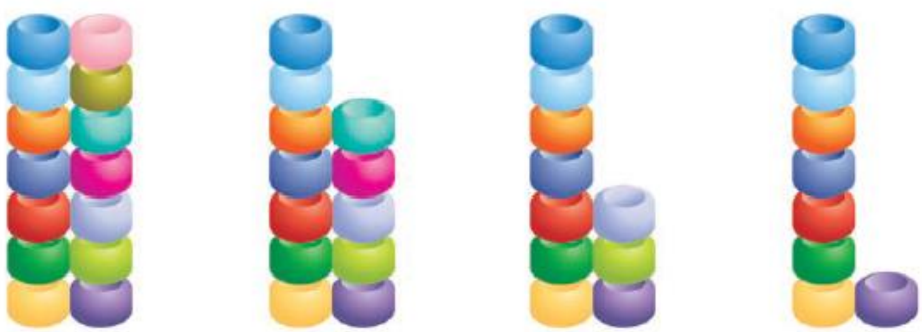
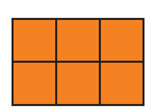


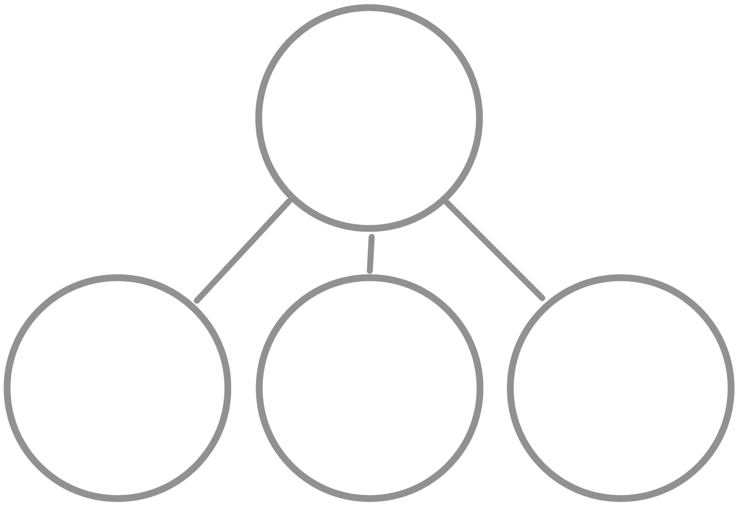
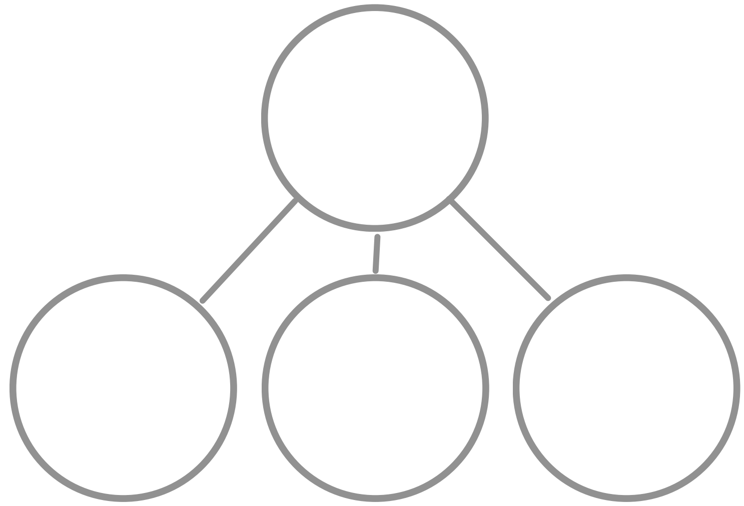
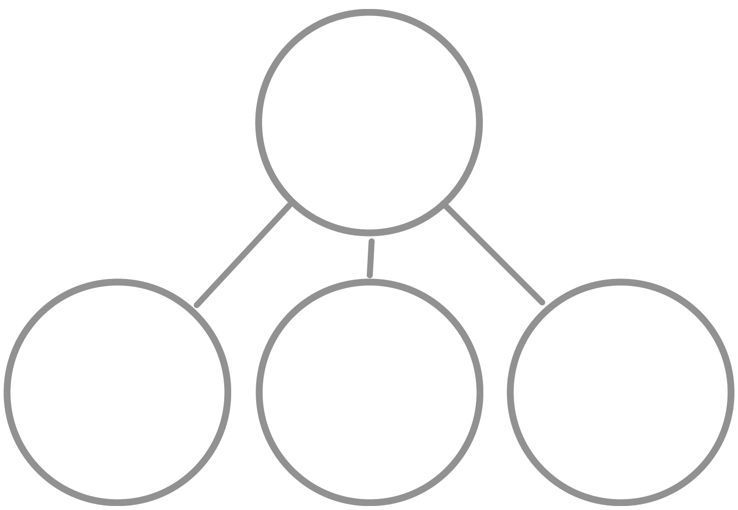
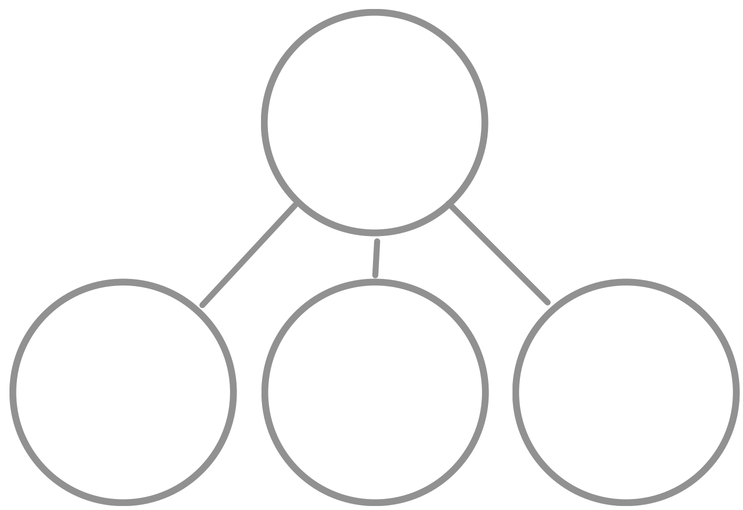
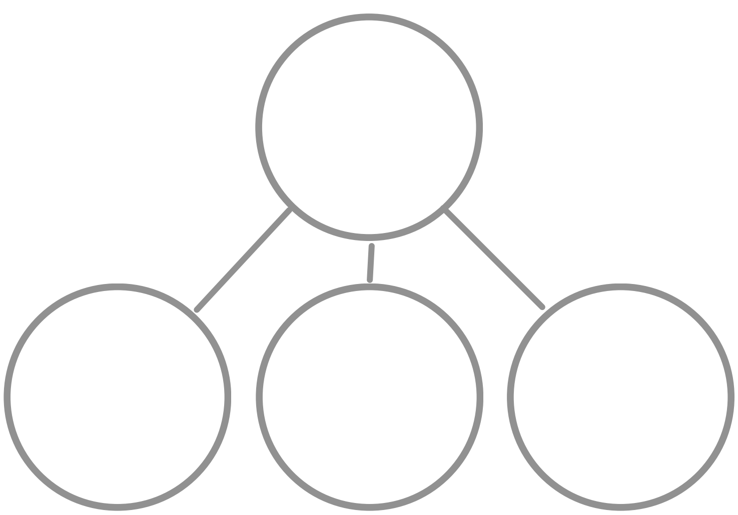
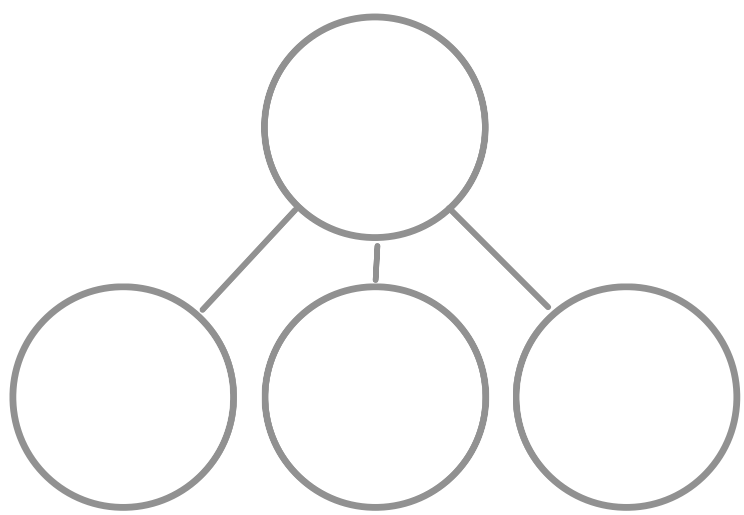
Figure 1 Figure 2 Figure 3 Figure 4

How many more figures can she make?

5. Draw the following pattern. Start with  and add one more than the last. Draw the next 3 figures.

Diamond Chart

Number Bond

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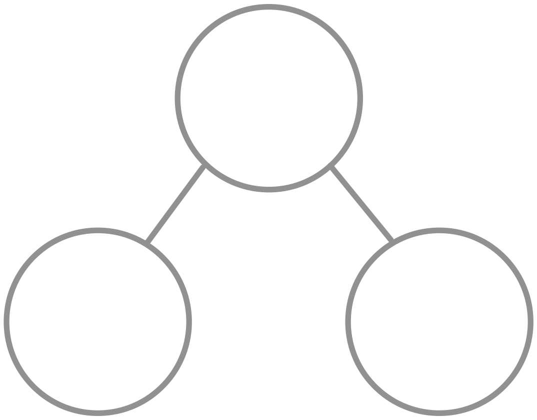
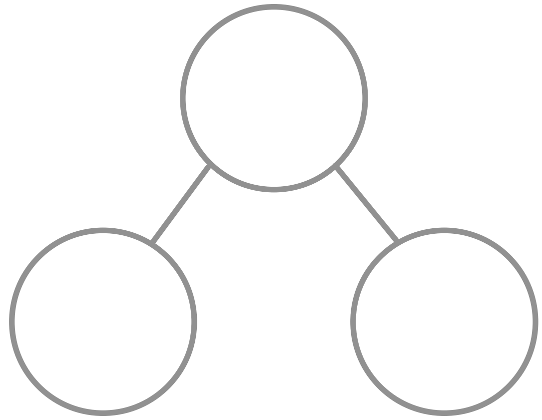

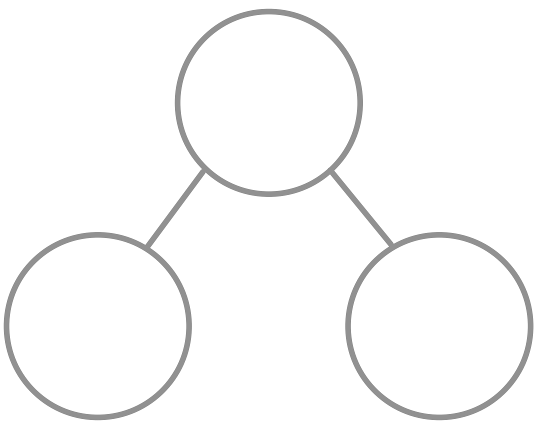
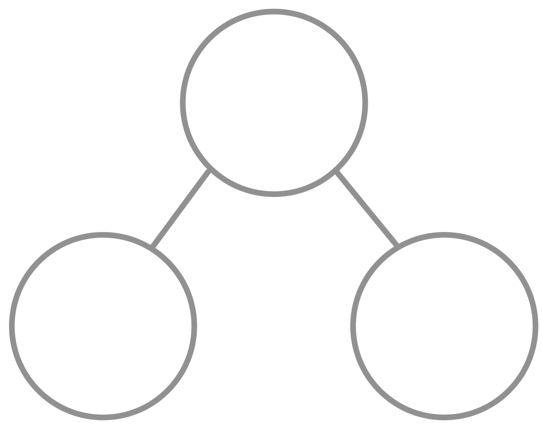

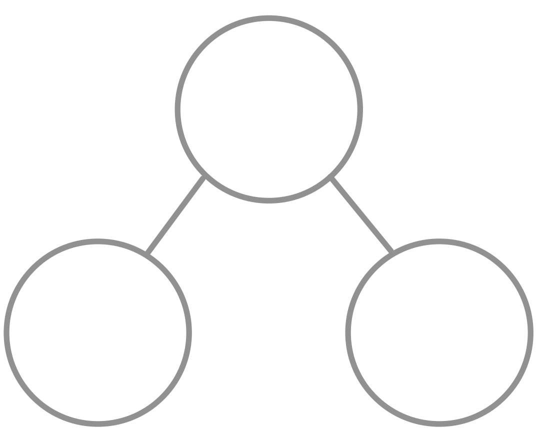
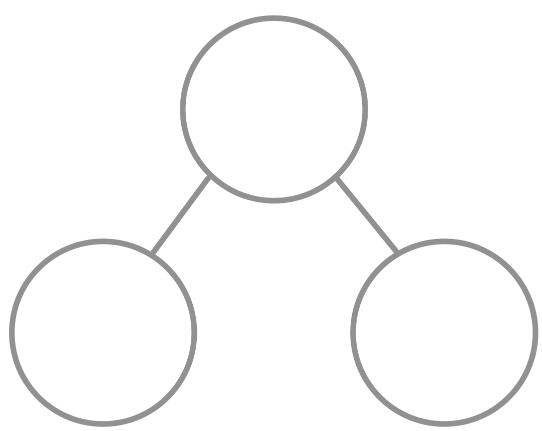

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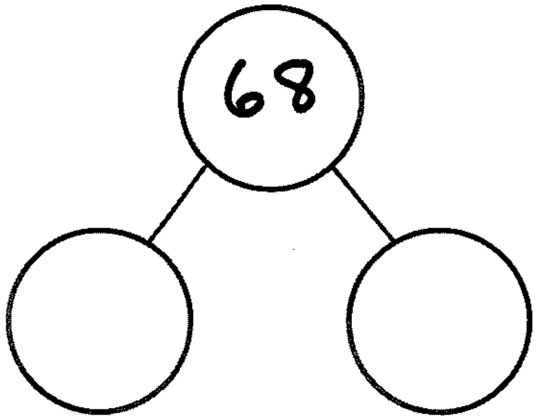

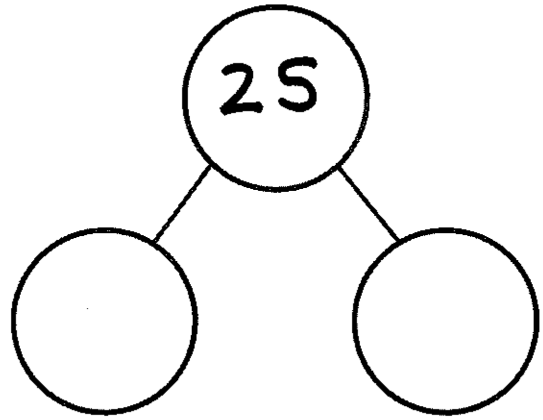

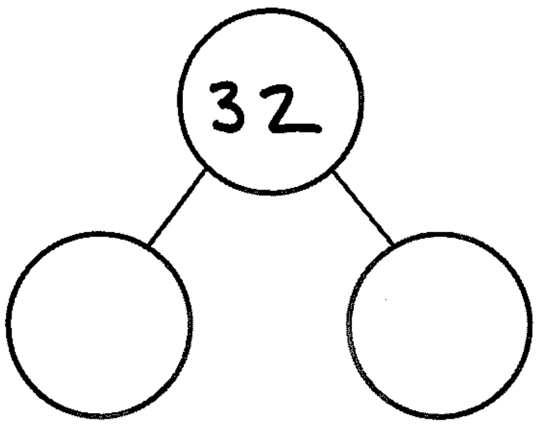

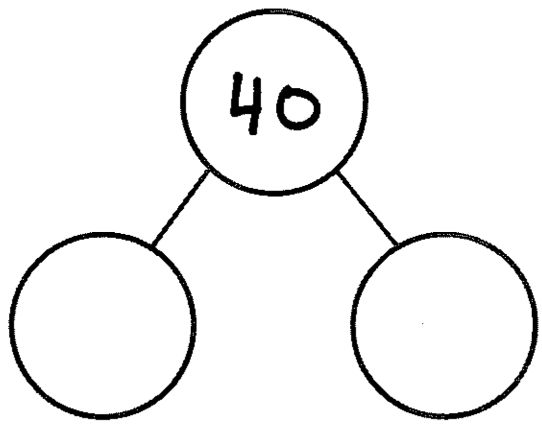

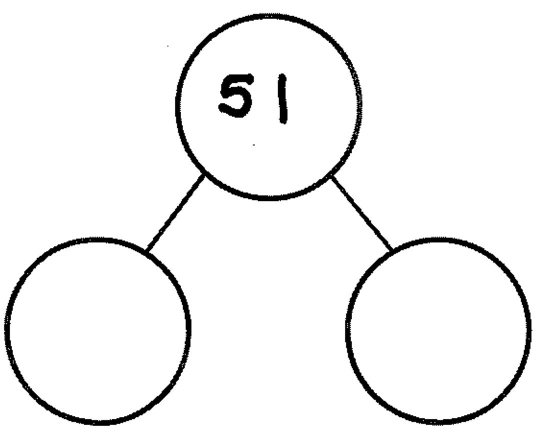

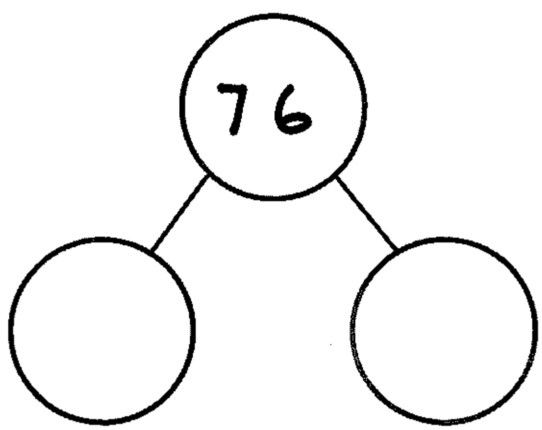

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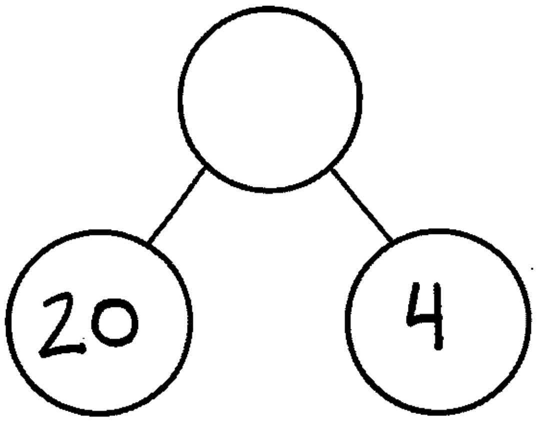

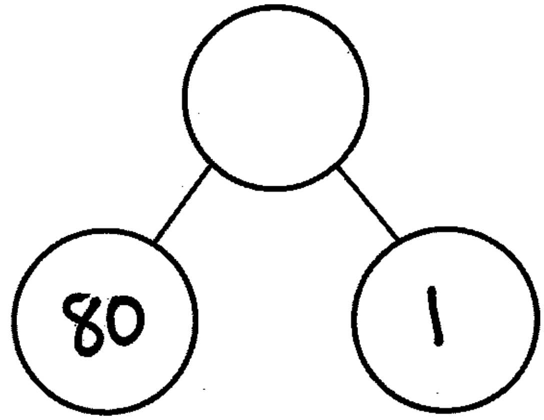

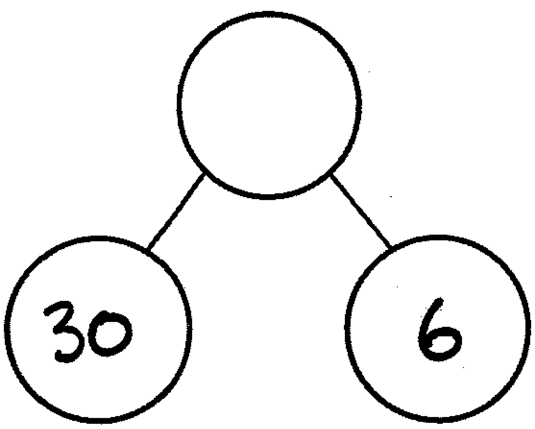

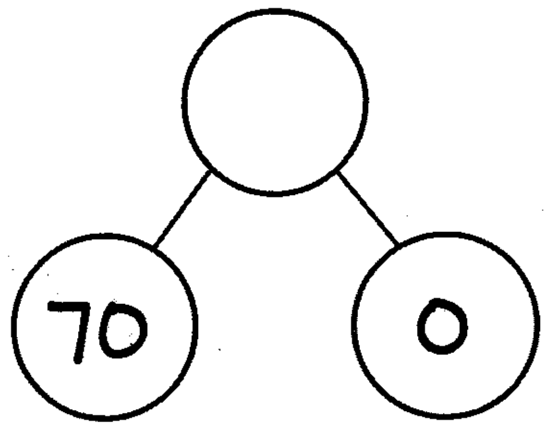

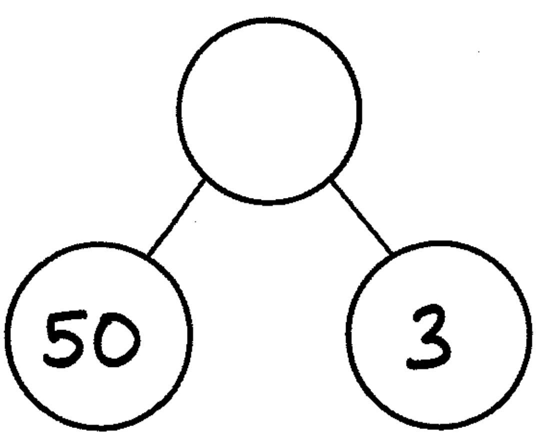

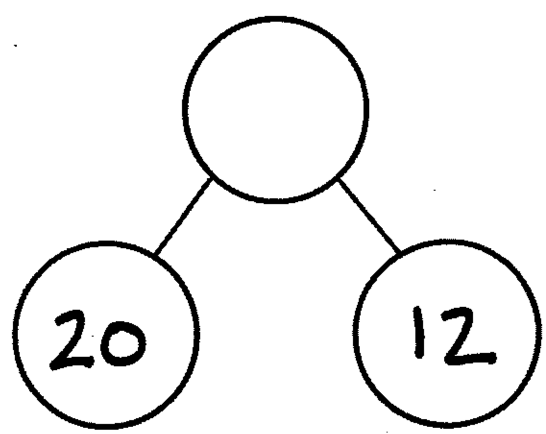

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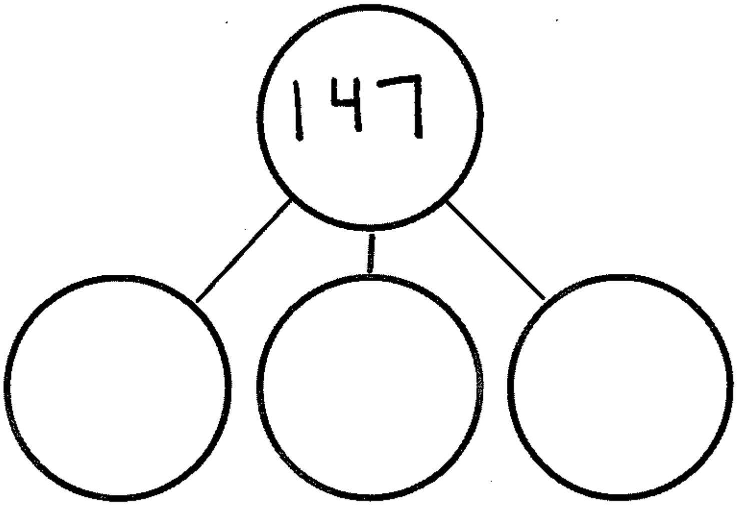
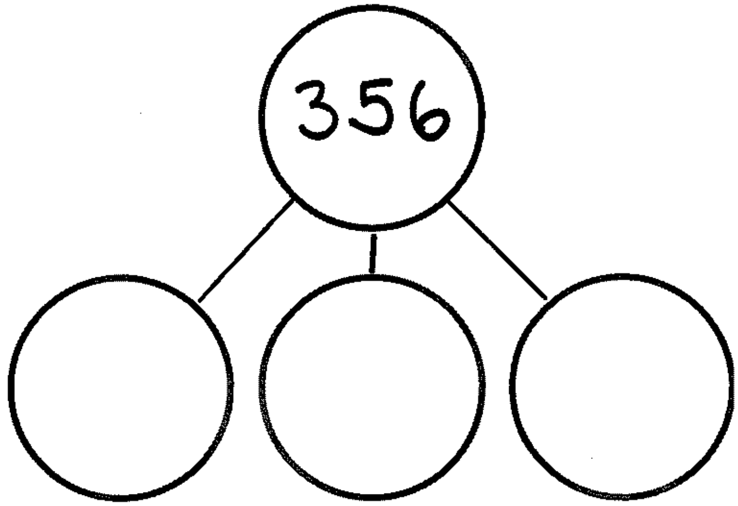
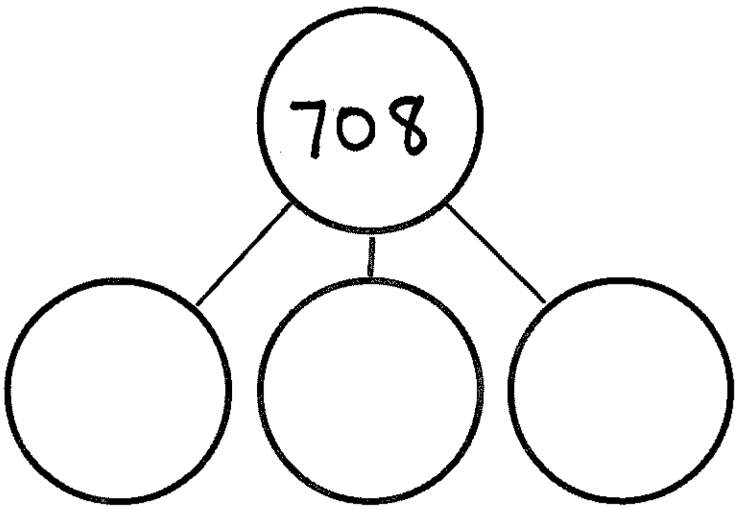
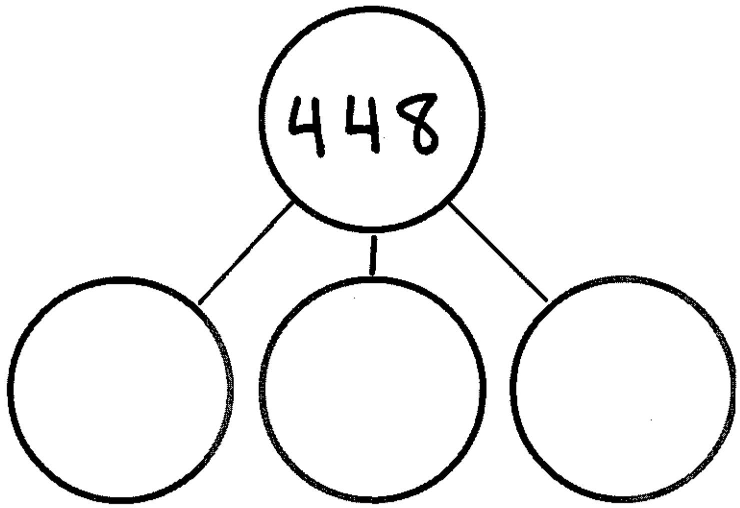
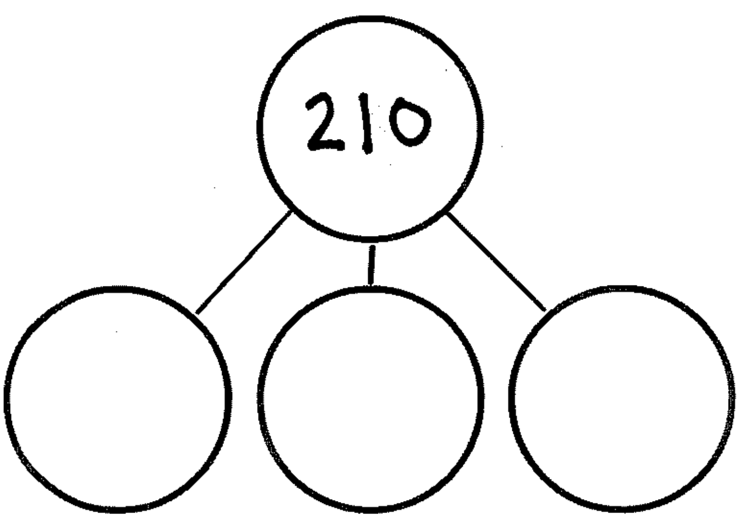
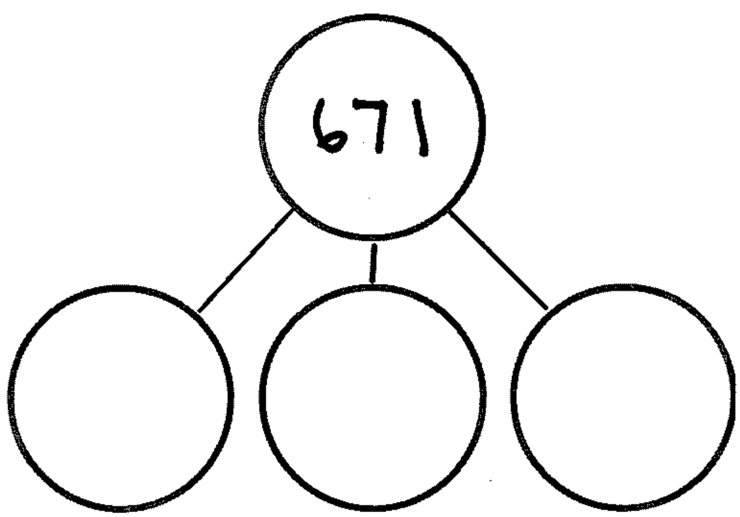












Number Bond

200 40 6

800 70 1

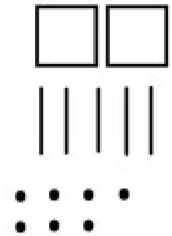
600 0 7

500 40 0

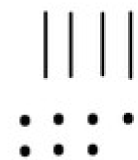
700 30 8

300 50 12

Name:____

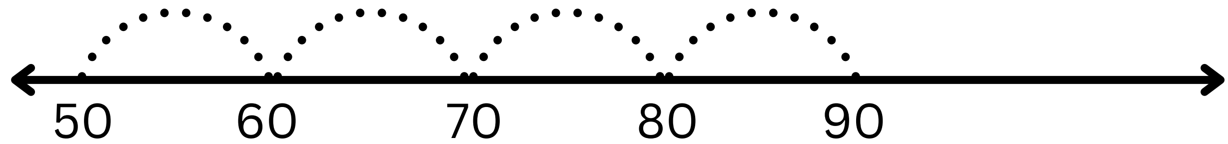
Standard Form	Expanded Form	Base Ten Blocks	Base Ten Name
			
	400 + 80 + 6		
143			
			7 hundred 16 tens 22 ones

Name:_______________________________________

Standard Form	Expanded Form	Base Ten Blocks	Base Ten Name
			
	80 + 6		
43			
			6 tens 2 ones

Number Line

50 + 40



50 + 40 = 90

30 + 30

10 + 30

20 + 70

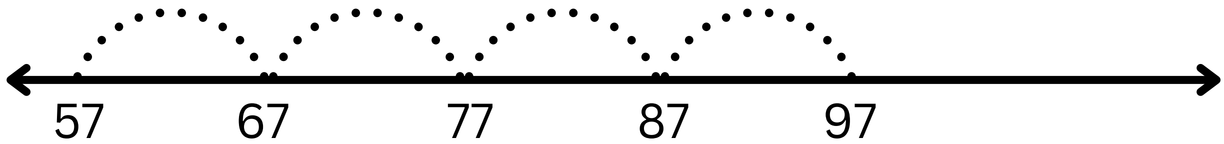
40 + 80

50 + 30

60 + 20

Number Line

57 + 40



57 + 40 = 97

34 + 30

14 + 30

28 + 70

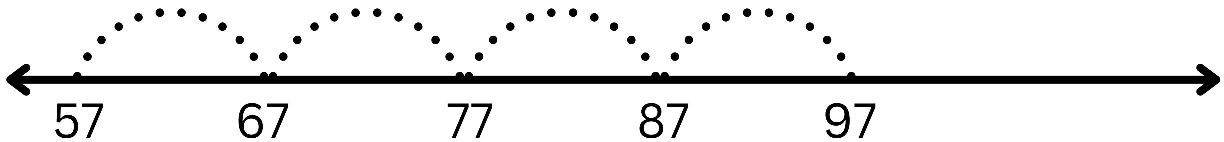
42 + 80

50 + 35

60 + 22

Number Line

57 + 40



57 + 40 = 97

34 + 30

134 + 30

134 + 100

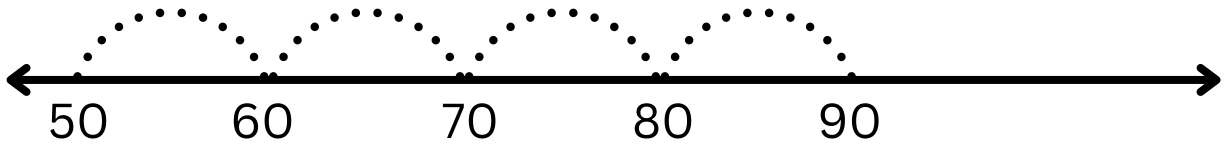
134 + 130

134 + 132

246 + 321

Number Line

90 - 40



90 - 40 = 50

60 - 30

40 - 30

90 - 70

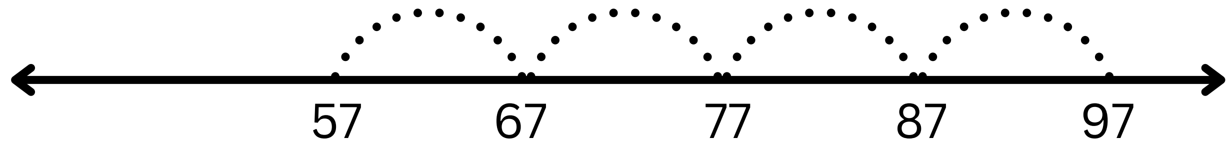
80 - 40

50 - 30

60 - 20

Number Line

97 - 40



97 - 40 = 57

64 - 30

36 - 10

88 - 50

92 - 80

85 - 60

61 - 20

Math Check In

3 tens =

3 + 1 =

30 + 10 =

3 tens + 4 tens =

5 tens =

7 tens =

2 tens =

9 tens =

50 + 10 =

30 + 20 =

30 + 40 =

20 + 80 =

32 + 10 =

46 + 20 =

55 + 40 =

Math Check In

Name: ______

1. 57

Draw #1

Draw #2

Break up #1

57

Break up #2

57

2. $\boxed{}$ → 40, 8

$\boxed{35}$

$\boxed{}$ → 20, 17

3.

10 + 5 = ______

20 + 8 = ______

70 + 2 = ______

60 + 8 = ______

40 + 2 = ______

50 + 9 = ______

Math Check In   Name: ________

346

Draw #1

Draw #2

Break up #1

346

Break up #2

346

2. $\boxed{}$ 500 30 7

$\boxed{478}$

$\boxed{}$ 400 60 13

3. 50 + 7 = ______

236 + 100 = ______

70 + 8 = ______

500 + 67 = ______

132 + 100 = ______

356 + 10 = ______

ADDITION

"Once someone learns a fast and efficient method for solving a problem, it's hard to work backward and explain why it works. So, instead we teach the ***why*** first, allowing kids to explore the math and determine rules and procedures for themselves."

Kreisberg & Beyranevand, *Adding Parents to the Equation*, 2019.

What is Addition?

Addition is the process of combining two or more things together.

Addend + Addend = Sum

Where is Addition in the Sask Curriculum?

Saskatchewan CURRICULUM Education: The future within us.

Grade One

Demonstrate an understanding of addition of numbers with answers to 20.

Grade Two

Demonstrate an understanding of addition (limited to 1 and 2-digit numbers) with answers to 100.

Grade Three

Demonstrate an understanding of addition of whole numbers with answers to 1000 (limited to 1, 2, and 3-digit numbers).

Grade Four

Demonstrate an understanding of addition of whole numbers with answers to 10 000 (limited to 3 and 4-digit numerals)

Grade Five

Demonstrate an understanding of addition of decimals (limited to thousandths).

Grade Six

Demonstrate understanding of the order of operation on whole numbers (excluding exponents) with and without technology.

What are the Common Properties of Addition?

COMMUTATIVE

The order of the addends **does not** matter when finding the sum.

If you count 5 buttons first, then three buttons it **is the same amount** as if you counted 3 buttons first then five buttons.

Ex) 5 + 3 = 3 + 5

ASSOSCIATIVE

The sum is the same regardless how the addends are grouped.

If you add 4 blocks and two blocks together, then add one more you get the same amount as if you add 2 blocks and 1 block together, then add 4 more blocks.

Ex) (4 + 2) + 1 = 4 + (2 + 1)

IDENTITY

The sum of any number and zero is the original number. Ex) $6 + 0 = 6$

Strategies vs Models

Strategies and models are not the same thing when solving a math problem. When we solve problems mentally, we need a way to show others how we solved the problem which we do through models.

A **strategy** is how you solve the problem.

A **model** is how you show the problem or your strategy.

For example, I may use a chunking strategy (see below) to add two numbers mentally and model my strategy on an open number line.

What are the Common Strategies of Addition?

The order and sequence of the following strategies is not how they should be introduced or instructed to children. Sharing the various strategies will help you identify the method in which your child might be solving problems in order to engage in conversation with your child confidently and comfortably about their strategy.

The addition strategies below should be taught through the Concrete - Representational – Abstract approach, which allows students to build conceptual understanding first through concrete manipulatives, then drawings and representations and finally with abstract numbers. Skipping these steps and moving quickly to rote memorization will result in students having procedural understanding of subtraction which may result in coming to the correct answer, however the student will be unable to be flexible and efficient in transferring their understanding to other problems.

Addition

There is no expectation that your child will use or learn all the strategies below but rather should be exposed to a variety of strategies that they understand and can use depending on the situation.

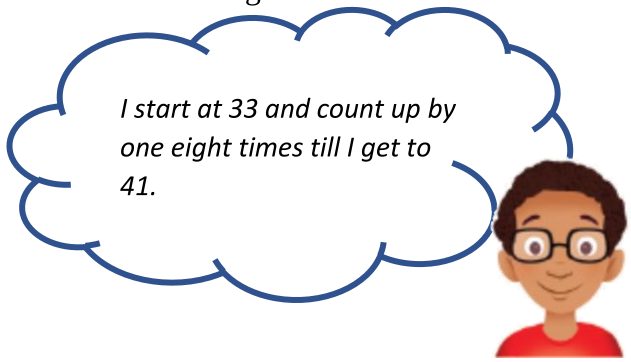
Single Digit Addition Strategies

Counting on

This strategy is primarily used in primary grades or with smaller amounts. Students will start from the first number and count on or students will start with the larger number and count on.

Example: 33 + 8

33…34, 35, 36, 37, 38, 39, 40, 41

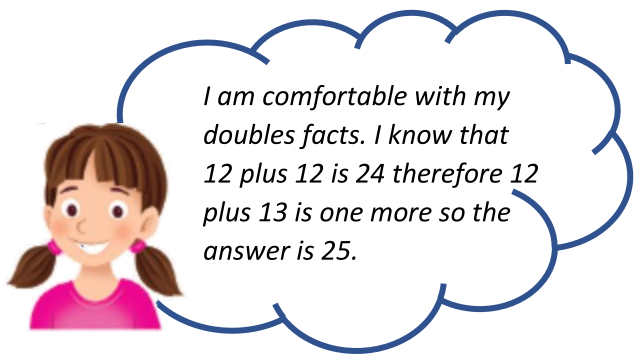


Doubles / Near Doubles

Students can recall sums for many doubles early on. Students build on this knowledge and adjust one or both numbers to make a doubles or near doubles combination.

Example: 12 + 13

12 + (12 + 1)
(12 + 12) + 1
24 + 1 = 25



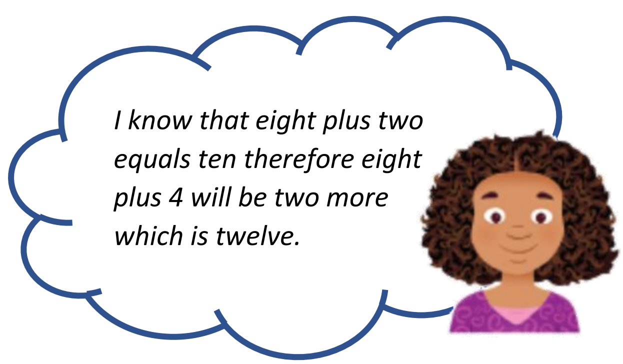
Known Facts

Students use their experience and knowledge of known facts that they can recall to help them solve other addition questions.

Example: 8 + 4

8 + 2 = 10

8 + 4 = 12



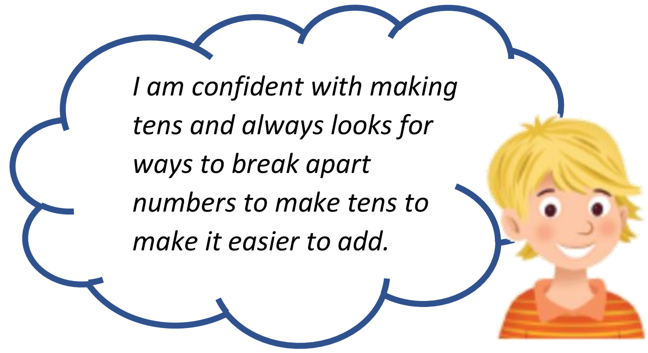
Multi-Digit Addition Strategies

Making Tens

Combination of numbers that make tens is an important concept for students in primary grades. Students should be able to break numbers apart to make tens with ease to use this strategy efficiently and flexibly.

Example: 116 + 118

(110 + 4 + 2) + (110 + 8)
110 + 110 + (2 + 8) + 4
110 + 110 + 10 + 4
230 + 4 = 234



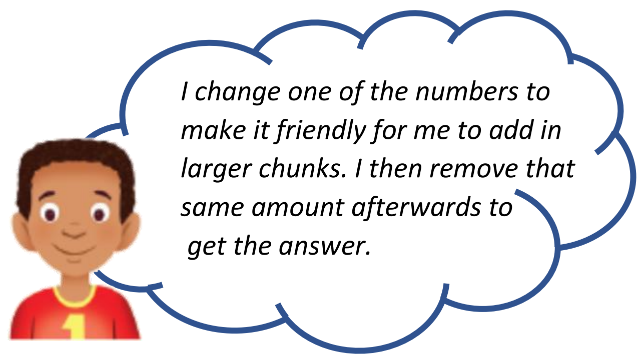
Landmark / Friendly Numbers

These are numbers that are easy to use in mental computation. These numbers may vary from student to student but are typically multiples of ten, hundred, and so on. Students may adjust one or both addends by adding or subtracting amounts to make a friendly number.

Example: 127 + 218

127 + 218
+ 2

127 + 220 = 327
327 – 2 = 325

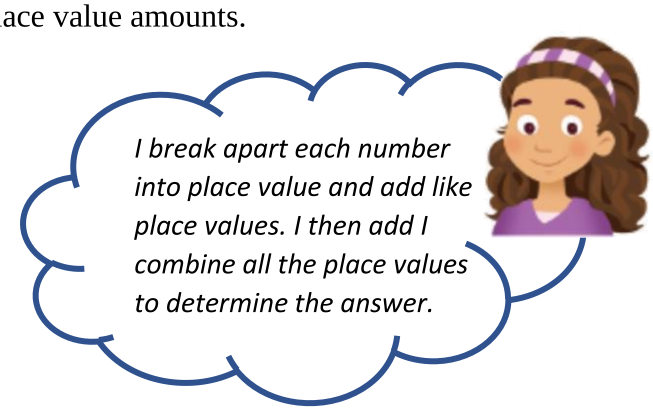


Place Value

Students will use this strategy when they begin to understand place value. Each added is broken up into expanded form and like place value amounts are combined. Students can work from left to right or right to left when combining like place value amounts.

Example: 237 + 148

(200 + 30 + 7) + (100 + 40 +8)
200 + 100 = 300
30 + 40 = 70
7 + 8 = 15
300 + 70 + 15 = 385



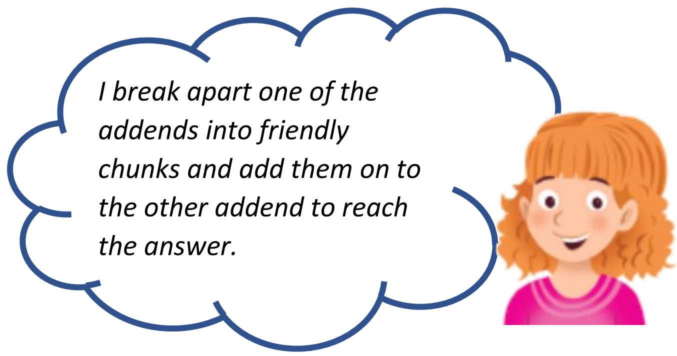
Addition

Adding up in Chunks

This is a similar strategy to Place Value except students keep one addend whole and break apart the second number into easy to use chunks (the chunks will vary based on student understanding).

Example: 126 + 58

126 + (50 + 4 + 4)
126 + 50 = 176
176 + 4 = 180
180 + 4 = 184



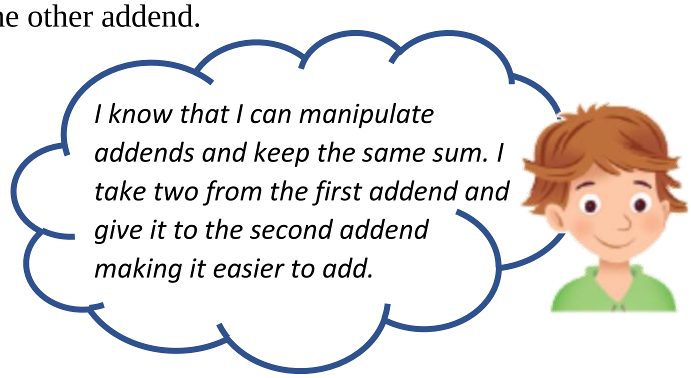
Compensation

This is a similar strategy to Friendly/Landmark Numbers. Students manipulate the numbers into easier, friendlier numbers to add. The difference is that students remove a specific amount from one addend and give that exact amount to the other addend.

Example: 237 + 148

- 2   + 2

235 + 150 = 385

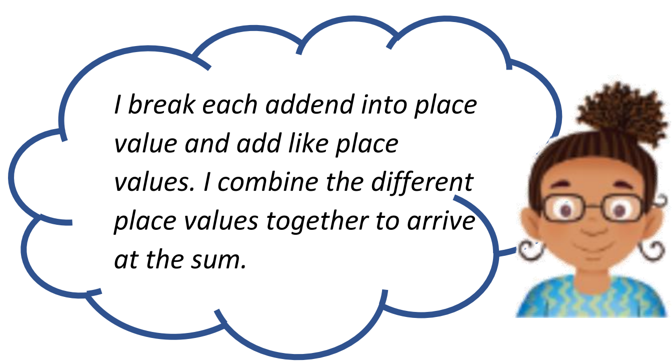


Expanded Algorithm 1

This strategy is similar to Place Value, except it is written in a different way as students move to a more efficient way of writing their thinking. This strategy explicitly shows that students understand place value of each digit in the addends.

Example: 137 + 256

$$\begin{array}{r} 100 + 30 + 7 \\ +\quad 200 + 50 + 6 \\ \hline 300 + 80 + 13 = 393 \end{array}$$

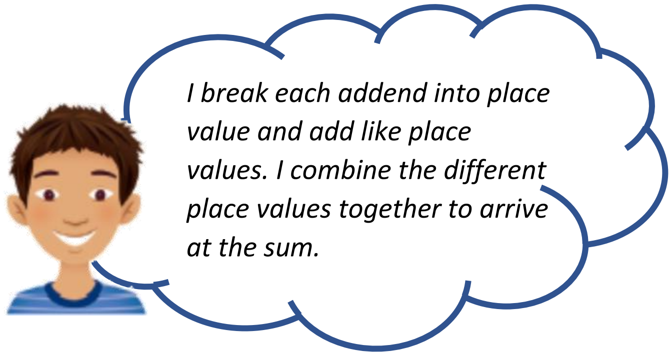


Expanded Algorithm 2

Students add like place value amounts. This strategy explicitly shows the addition of each like place value. Students then add up the remaining amounts and the ways they add them up may differ depending on amounts and understanding.

Example: 137 + 256

$$\begin{array}{r} 137 \\ +\,\underline{256} \\ 300 \\ 80 \\ \underline{13} \\ 393 \end{array}$$



Standard Algorithm

Students work from the right to the left adding each like place value amount and efficiently notes what they are doing. Students may turn multi-digit addition into single digit addition for each place value amount. The difference here is that students understand what they are doing throughout the algorithm as they have come to this strategy with understanding as they constructed their knowledge of other strategies. When asked to explain their strategy, they can explain using correct language and understanding.

Example: 137 + 256

$$\begin{array}{r} {\scriptstyle 1} \\ 137 \\ +\,\underline{256} \\ 393 \end{array}$$

SUBTRACTION

"It is very hard for us adults to not equate familiarity with understanding. We are very familiar with it [standard algorithm], therefore we feel we understand it, but imagine teaching it for the very first time, that is very, very hard"

James Tanton, Mathematician

What is Subtraction?

Subtraction is the "inverse" of addition and it undoes what addition accomplishes. Subtraction is finding the difference between two quantities. Subtraction can be interpreted the following ways: 1) take away, 2) comparison, and 3) the distance between two things or missing addend.

Minuend - Subtrahend = Difference

Saskatchewan CURRICULUM Education: The future within us.

Grade One

Demonstrate an understanding of subtraction of whole numbers under 20.

Grade Two

Demonstrate an understanding of subtraction (limited to 1 and 2-digit numbers) of whole numbers under 100.

Grade Three

Demonstrate an understanding of addition of whole numbers (limited to 1, 2, and 3-digit numbers) under 1000.

Grade Four

Demonstrate an understanding of subtraction of whole numbers (limited to 3 and 4-digit numbers) under 10 000.

Grade Five

Demonstrate an understanding of subtraction with decimals to the thousandths.

Grade Six

Demonstrate understanding of the order of operation on whole numbers (excluding exponents) with and without technology.

What are the Common Properties of Subtraction?

COMMUTATIVE	ASSOSCIATIVE
The order **does** matter when you subtract. If you have 5 buttons and remove 3 buttons **it is not the same** amount as if you had 3 buttons and removed 5 buttons. Ex) $5 - 3 \neq 3 - 5$	The difference **is not the same** regardless how the quantities are grouped. Ex) $(4 - 2) - 1 \neq 4 - (2 - 1)$ $(3) - 1 \neq 4 - (1)$ $2 \neq 3$

Strategies Vs Models

Strategies and models are not the same thing when solving a math problem. When we solve problems mentally, we need a way to show others how we solved the problem which we do through models.

A **strategy** is how you solve the problem.

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For example, I may use a chunking strategy (see below) to add two numbers mentally and model my strategy on an open number line.

What are the Common Strategies of Subtraction?

The order and sequence of the following strategies is not how they should be introduced or instructed to children. Sharing the various strategies will help you identify the method in which your child might be solving problems in order to engage in conversation with your child confidently and comfortably about their strategy.

The subtraction strategies below should be taught through the Concrete - Representational – Abstract approach, which allows students to build conceptual understanding first through concrete manipulatives, then drawings and representations and finally with abstract numbers. Skipping these steps and moving quickly to rote memorization will result in students having procedural understanding of subtraction which may result in coming to the correct answer, however the student will be unable to be flexible and efficient in transferring their understanding to other problems.

There is no expectation that your child will use or learn all the strategies below but rather should be exposed to a variety of strategies that they understand and can use depending on the situation.

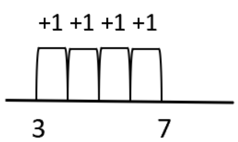
Subtraction

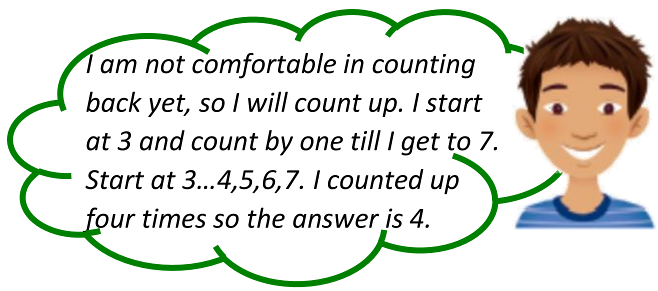
Single Digit Subtraction

Adding Up

Students build on their strength of addition and add up from the subtrahend the number being subtracted) to the minuend (the whole) to find the difference between the two numbers. Students will jump to the nearest ten or friendly numbers and the larger the jumps the more efficient the strategy will be.

Example: 7 – 3 = 4

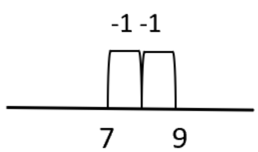


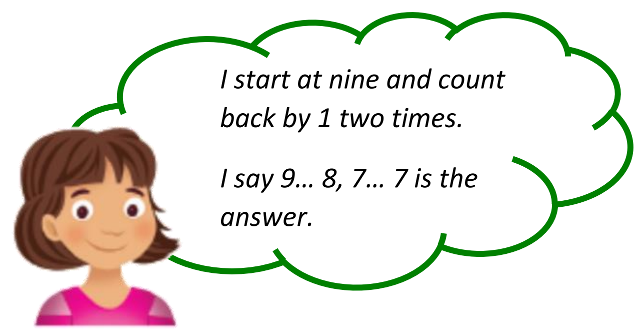


Removal / Counting Back

Students can decompose the subtrahend into easy to remove parts and take these jumps backwards from the minuend. The bigger the jumps the more efficient the strategy will be.

Example: 9 – 2 = 7



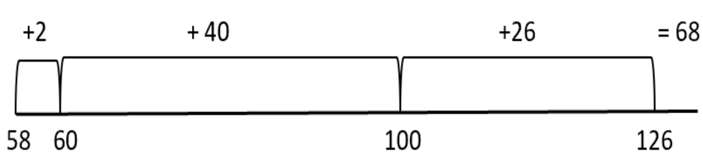


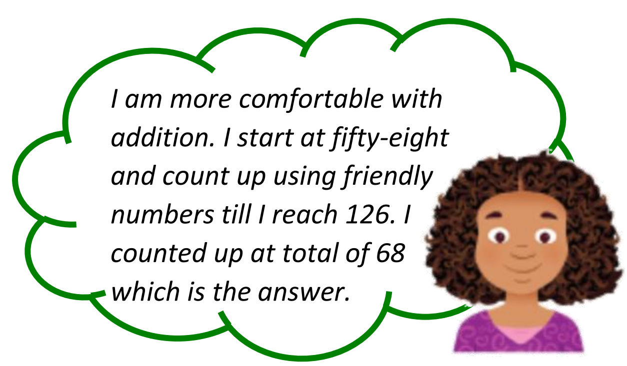
Multi-Digit Subtraction

Adding Up

Students build on their strength of addition and add up from the subtrahend the number being subtracted) to the minuend (the whole) to find the difference between the two numbers. Students will jump to the nearest ten or friendly numbers and the larger the jumps the more efficient the strategy will be.

Example:126-58





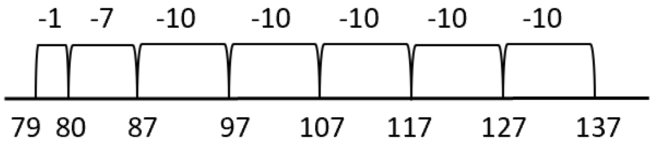
Subtraction

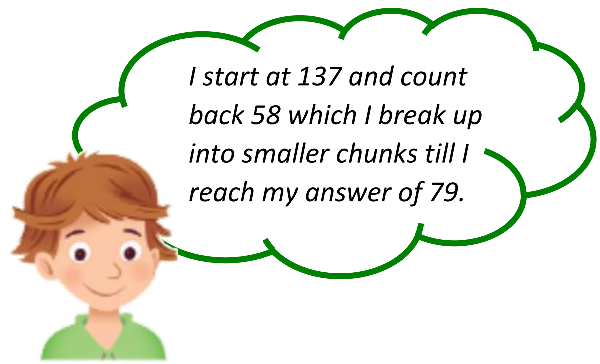
Removal / Counting Back

Students can decompose the subtrahend into easy to remove parts and take these jumps backwards from the minuend. The bigger the jumps the more efficient the strategy will be.

Example:137 – 58

137 – (10 + 10 + 10 + 10 + 10 + 7 + 1)





Place Value & Negative Numbers

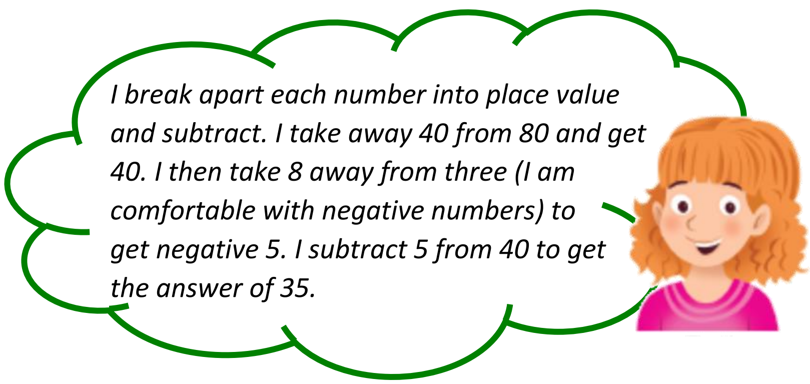
Students break each number into place value. Like place values are grouped and then subtracted, negative values might result.

Example:83 – 48

(80 + 3) – (40 – 8)

$$\begin{array}{rrr} & 80 & 3 \\ - & 40 & 8 \\ \hline & 40 & -5 \end{array}$$

$$40 - 5 = 35$$

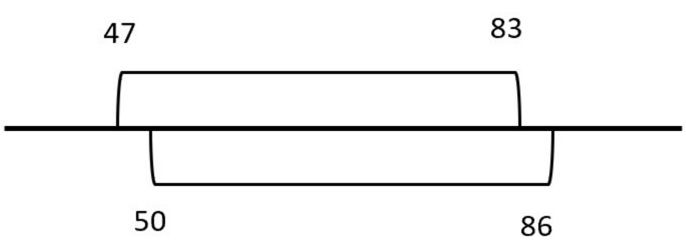


Constant Difference

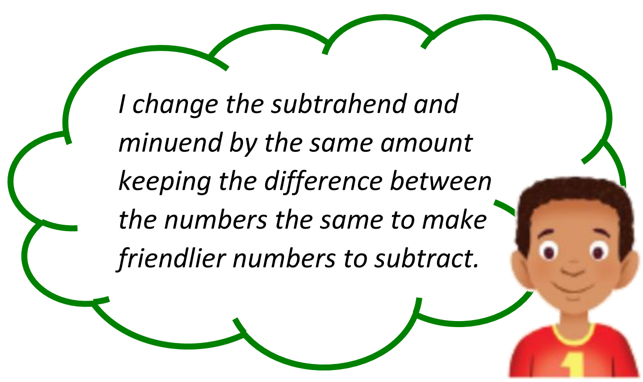
As students become more familiar with subtraction and begin to understand subtractions as the difference between quantities, they can add or subtract the same amount from both numbers and keep the same difference. Manipulating the numbers allows students to create friendlier problems.

Example: 83 – 47

47 + 3 = 50 83 + 3 = 86



$$\mathbf{86 - 50 = 36}$$



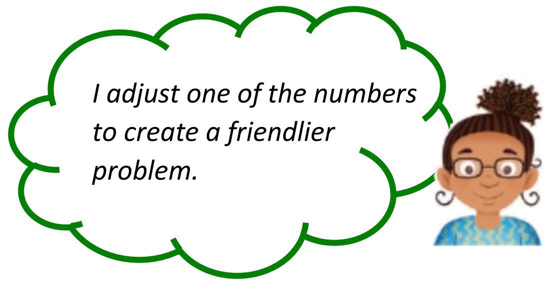
Subtraction

Adjusting One Number

Adjusting one of the numbers can turn the subtraction problem into a friendlier one. The decision is what number you are going to adjust to create a friendlier problem. The student's understanding of part/whole relationships will help them reason through these decisions.

Example: $164 - \underset{+2}{28}$

$$164 - 30 = 134$$
$$\underline{+2}$$
$$= 136$$

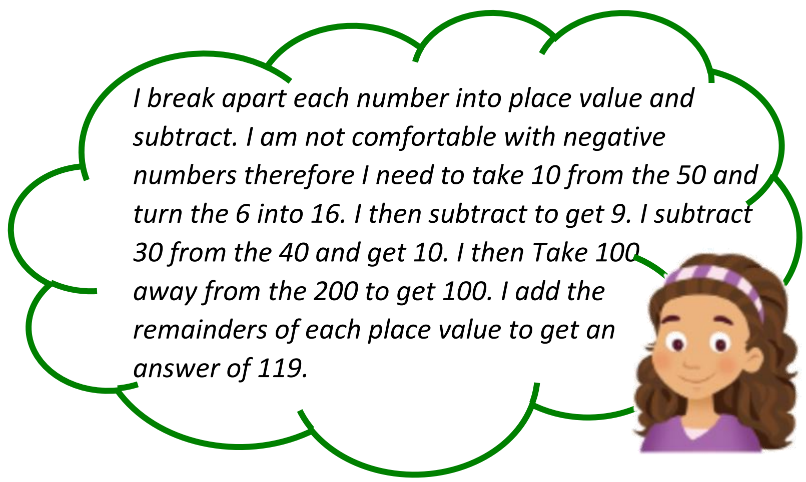


Expanded Algorithm 1

This strategy is like Place Value, except it is written in a different way as students move to a more efficient way of writing their thinking. This strategy explicitly shows that students understand place value of each digit in the minuend and subtrahend.

Example: 256 – 137

$$\begin{array}{rrrrrr} & 200 & + & \overset{40}{\cancel{50}} & + & {}^{1}6 \\ - & 100 & + & 30 & + & 7 \\ \hline & 100 & + & 10 & + & 9 \end{array}$$

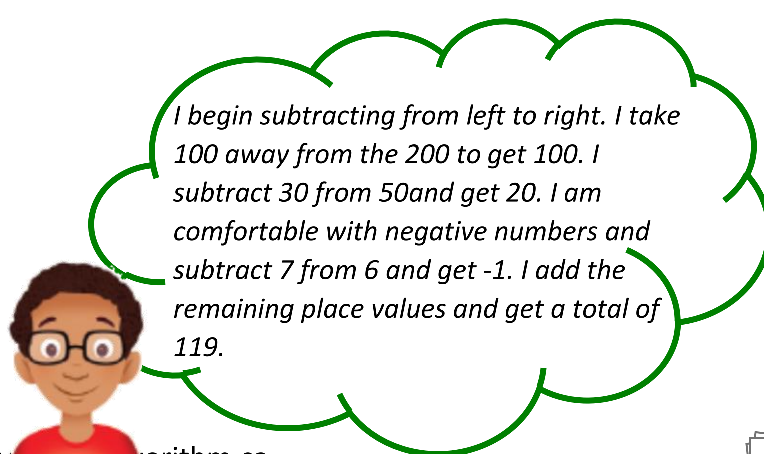


Expanded Algorithm 2

Students subtract like place value amounts. This strategy explicitly shows the subtraction of each like place value. Students then add up the remaining amounts and the ways they add them up may differ depending on amounts and understanding. Students who understand and are comfortable using negative numbers will be more efficient than those who must ungroup.

Example: 256 – 137

$$\begin{array}{rrrr} & 2 & 5 & 6 \\ - & 1 & 3 & 7 \\ \hline & 1 & 0 & 0 \\ & & 2 & 0 \\ & & & -1 \\ \hline & 1 & 1 & 9 \end{array}$$



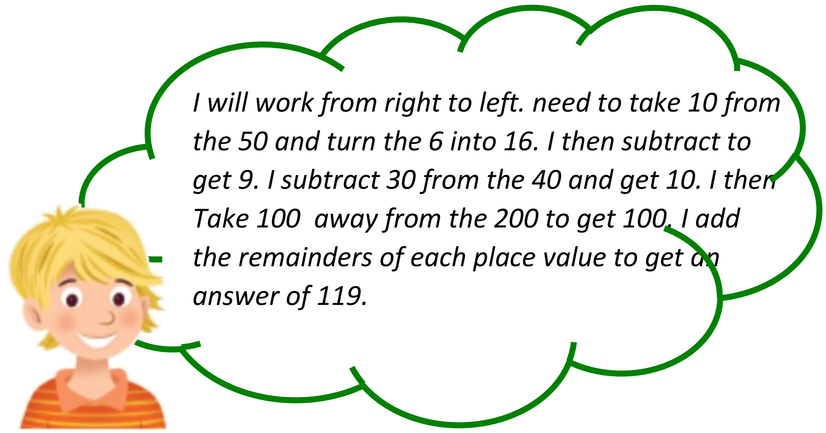
Subtraction

Standard Algorithm

Students work from the right to the left subtracting each like place value amount and efficiently notes what they are doing. Students may turn multi-digit subtraction into single digit subtraction for each place value amount. The difference here is that students understand what they are doing throughout the algorithm as they have come to this strategy with understanding as they constructed their knowledge of other strategies. When asked to explain their strategy, they can explain using correct language and understanding.

Example:256 – 137

$$
\begin{array}{rccc}
 & & 4 & \\
 & 2 & \cancel{5} & {}^{1}6 \\
- & 1 & 3 & 7 \\
\hline
 & 1 & 1 & 9
\end{array}
$$



I want to reemphasize that the standard algorithm is not the be it end all strategy we are hoping students get to solve all math problems. We want students to be able to use an efficient personal strategy that lends itself to the problem that is presented.

Reflection

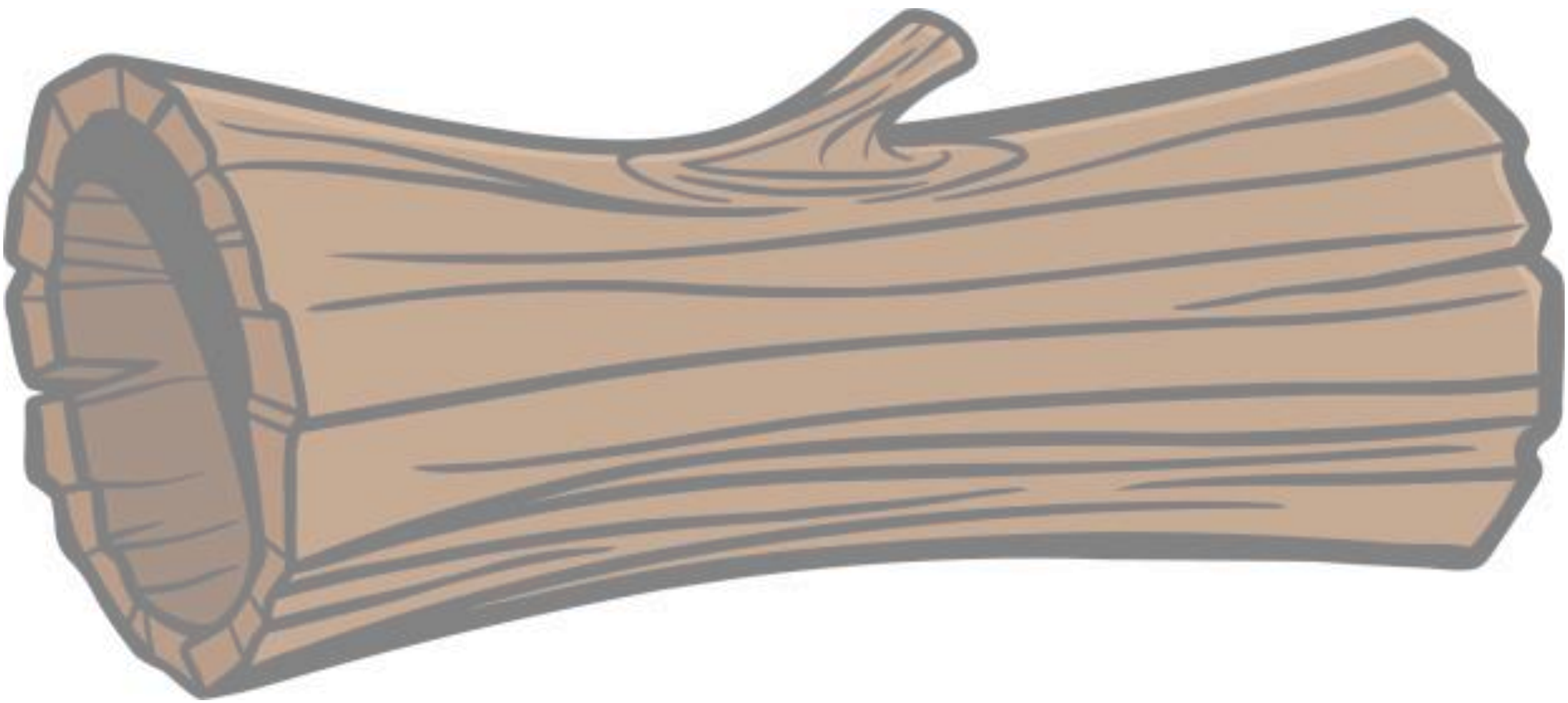
Think about how you would solve this problem and how you would hope your child solves the following problem. Would it be an algorithm? Or having the number sense to solve it in a more efficient way?

1000 – 998 =

Start

Change

Result



Numberless Word Problems: Log