



LEVERAGING STUDENT STRATEGIES

Chad Williams

Slides: Handouts:

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Welcome

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HOW DO YOU PROGRESS TO ADDITION & SUBTRACTION?

4

LEARNING TRAJECTORY	EQUALITY	MODELS VS STRATEGIES
COUNTING PRINCIPLES	WAYS TO SHOW NUMBERS	ADDITION & SUBTRACTION
- Stable order - Order irrelevance - Conservation - Subitizing - One-to-one - Cardinality - Hierarchical Inclusion - Utilizing	- Standard form - Build It - Base Ten Blocks - Base Ten Name - Expanded Form - Place Value	- Meaning - Relationships - Ways to construct knowledge - Children's inventions - Strategies that have legs - Properties - Problem types - Reasonableness

5

OUR JOURNEY TOGETHER

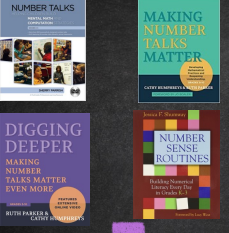
- What is Number Talks
- The Structure
- Tweak
- Extension
- Questions



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Number Talks



Classroom number talks provide a collaboration forum for students to:

- Invent strategies
- Generalize individual strategies into personal algorithms
- Build a conceptual bridge to the standard algorithm.



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Number Talks

→ Students are presented with problems in either a whole- or small-group setting and are expected to learn to mentally solve them accurately, efficiently, and flexibly.

→ By sharing and defending their solutions and strategies, students are provided with opportunities to collectively reason about numbers while building connections to key conceptual ideas in mathematics. (Parrish 2014)



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Structure

1. Write the question on the board (horizontally)
2. Allow student think time – Hand signals
3. Ask for possible solutions
4. Students defend a solution
5. Students share different strategies
6. Repeat for next 3 questions in string

Number Talks Hand Signals



I'm thinking.

I have an answer and a strategy.

I agree.

I have more than one strategy.

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Computational Fluency

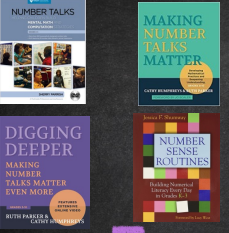
"Computational fluency refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently."

Principal and Standards for School Mathematics, NCTM, Reston, VA 2000, p.152



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Number Talks



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- Generalize individual strategies into personal algorithms
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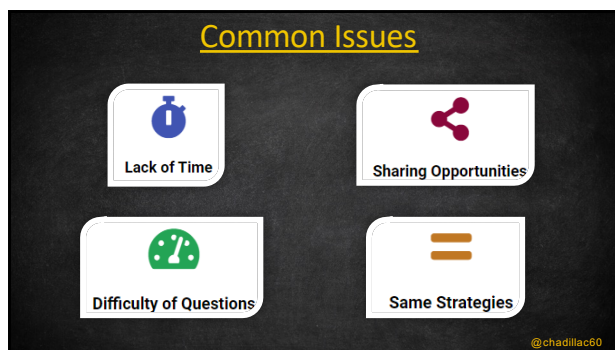
Key Components of Number Talks

- Classroom Environment
- Classroom Discussions
- The Teacher's Role
- The Role of Mental Math
- Purposeful Computation

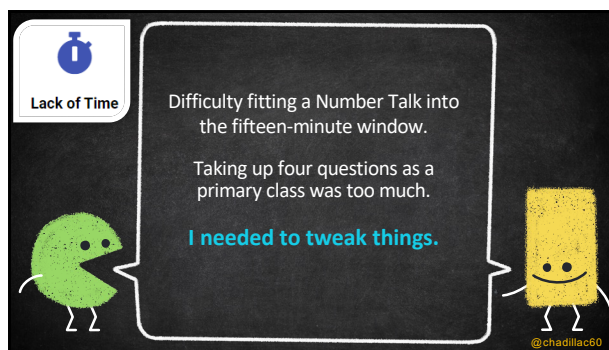
Parrish, 2014, P.10



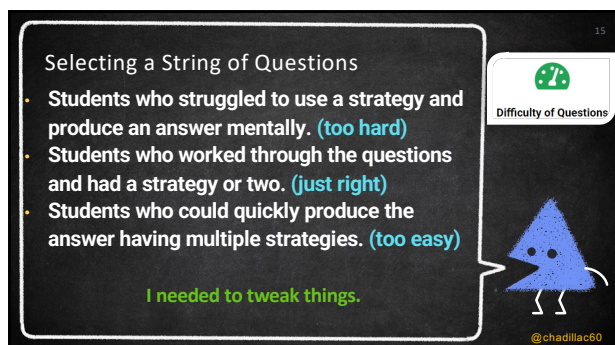
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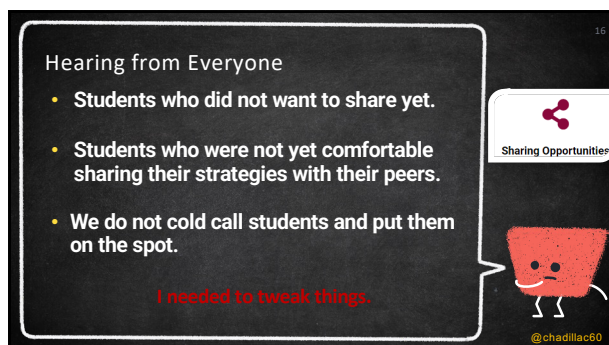
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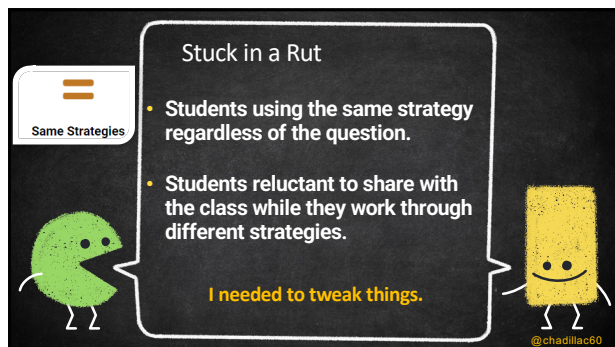
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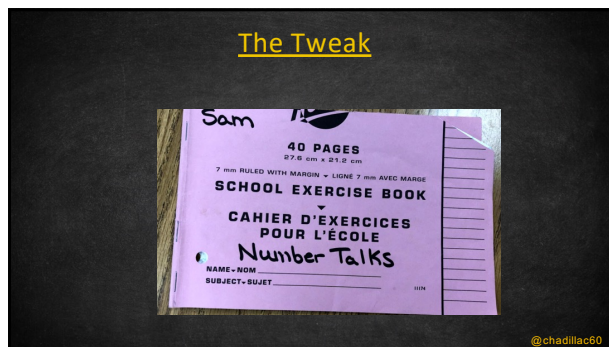
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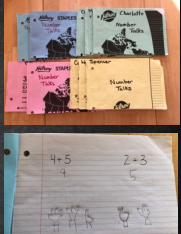
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Tweak to the Structure

- As a class we do two questions on the board
- Students do two questions in the Number Talk Booklets
- Color coding makes it quicker and easier for students to grab them off of the table.
- Five minutes set (timer or keep track myself)
- Students hand in their booklets when they are finished
- Begin working on open ended task or get themselves ready for next lesson.




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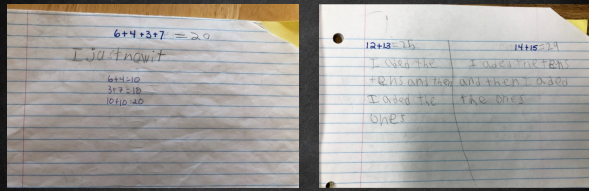
The Number Talk Booklet

- Allows students to share strategies daily
- Ensures each students had 2 questions at their level
- Quickly see how students are solving particular questions
- Information given to plan next steps
- Shows growth and learning journey
- Nudge students in their thinking and different strategies
- Builds confidence in students to share
- A product of their thinking and learning



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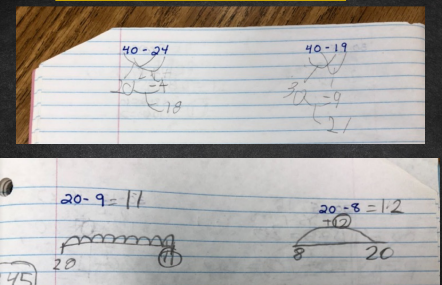
Share Strategies



"The Number Talks booklet gives me a chance to share my strategy if I don't get to in class"

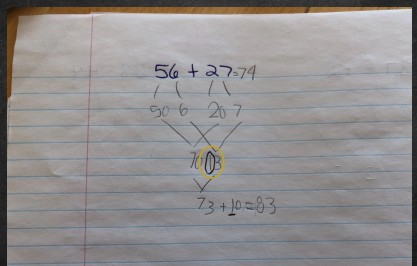
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Personalized Questions



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A Glance at Strategies



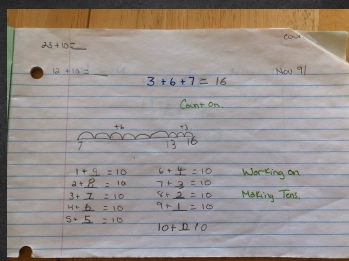
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Vital Information

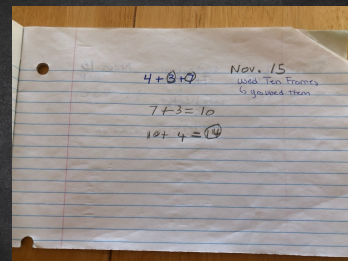
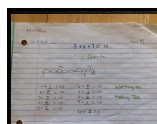
Name	23 + 15	56 + 27	26 + 58
Jaslyn	Count on	BTB	----
Precious	PV	*PV	PV
Jerome	Chunk	Comp	BA

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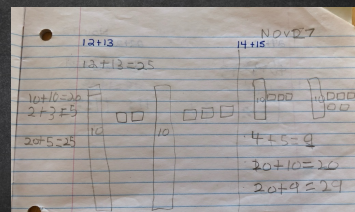
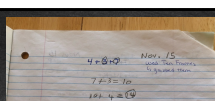
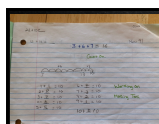
The Learning Journey



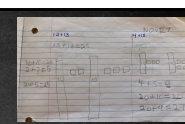
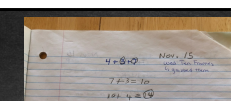
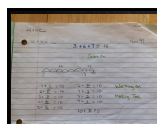
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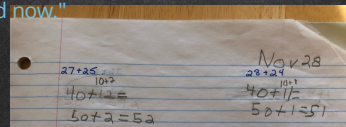
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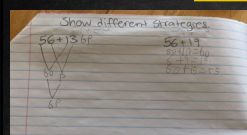


"I remember when I used to solve questions like that, but now it is easier. I get it! Math isn't that hard now."

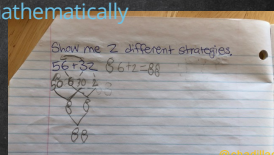


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Nudge Students

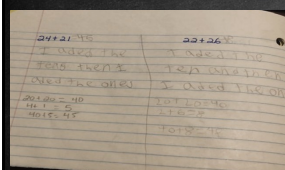


- To try different strategies
- Look at more efficient strategies
- How they write it mathematically



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Build Confidence



- In writing mathematically
- To share their strategies
- As young mathematicians

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Student Product

Handwritten student work showing multiplication problems and a triangle diagram. The problems include:

- $40 \div 2 = 20$
- $24 \div 6 = 4$
- $30 \div 5 = 6$
- $40 \div 10 = 4$
- $18 \div 9 = 2$
- $20 \div 2 = 10$
- $20 \div 10 = 2$
- $21 \div 7 = 3$

A triangle diagram with vertices labeled: Conversations (top), Products (bottom left), and Observations (bottom right).

Handwritten notes include: "Wed Nov 30", "167 + 123", "115 + 213", "167 + 100 = 267", "115 + 200 = 315", "267 + 70 = 337", "315 + 90 = 405", "337 + 3 = 340", "405 + 3 = 408".

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The Extension

Two photos of students working at whiteboards. The left photo shows a student writing on a whiteboard. The right photo shows two students working together at a whiteboard, with one student pointing at the board.

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"I really didn't want to miss yesterday; my brother is excited for me to tell him how it felt."
Liam, Grade 2

Name: _____

Number Talk

What are the two questions you will work through?

What do you want me to look for?

Any questions you have before you give it a try?

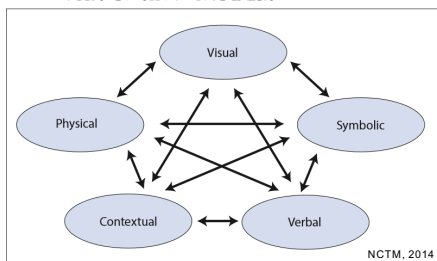
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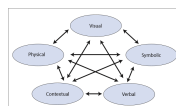
VEED.IO

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WAYS TO SHOW NUMBERS



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Students must construct, understand and use the structural similarities between any representation in order to use as tools for thinking.

Representations move from models of thinking to models for thinking.

Models are representations of relationships that mathematicians have constructed over time.

Models are **mental maps** used by mathematicians as they organize their activity, solve problems, and explore relationships.

Fosnot, 2001

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Big Idea:

Numbers can be broken up and put back together!

25

25

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Big Idea:

Numbers can be broken up and put back together!

20

5

2 Tens
5 Ones

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40

Big Idea:

Numbers can be broken up and put back together!

25

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Big Idea:

Numbers can be broken up and put back together!

25

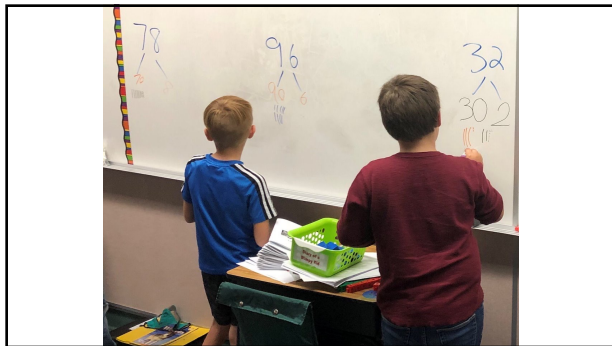
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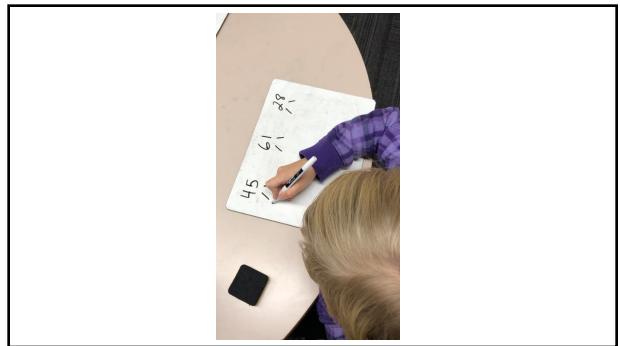
1 Tens
15 Ones

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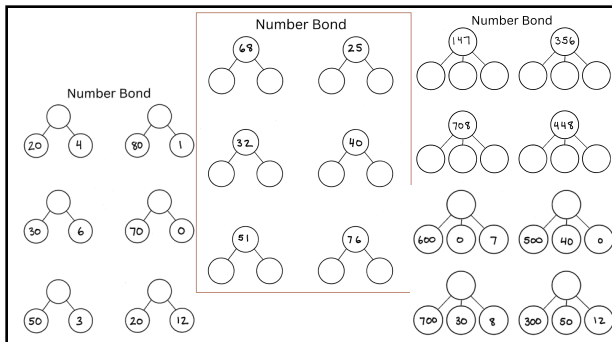
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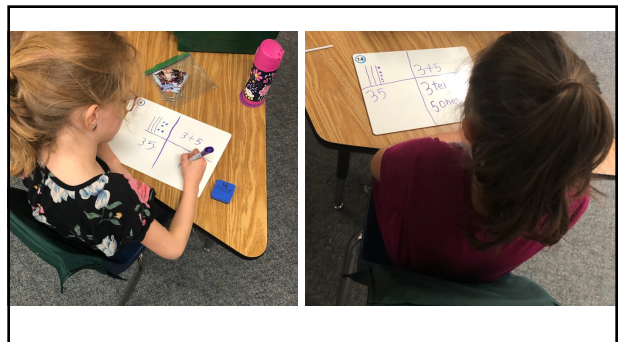
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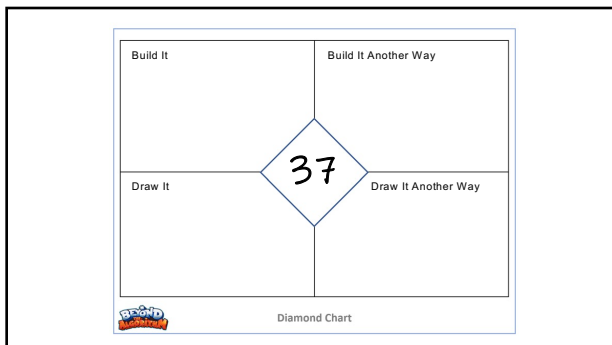
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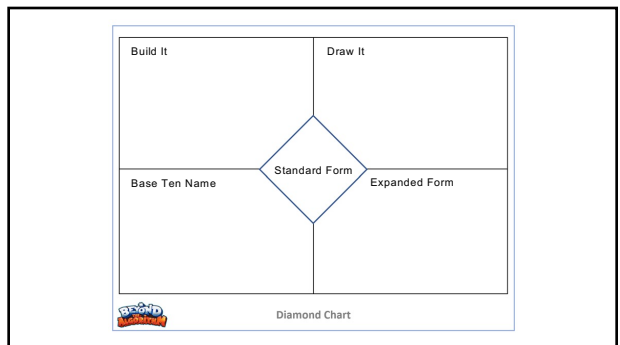
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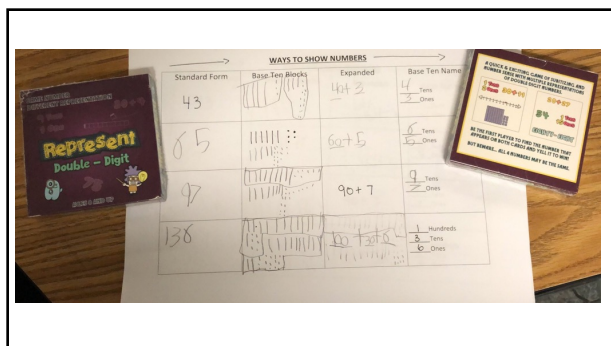
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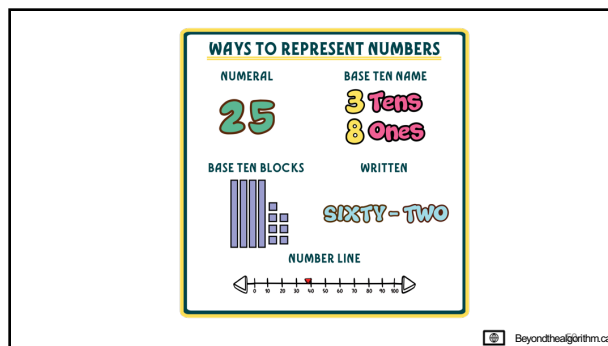
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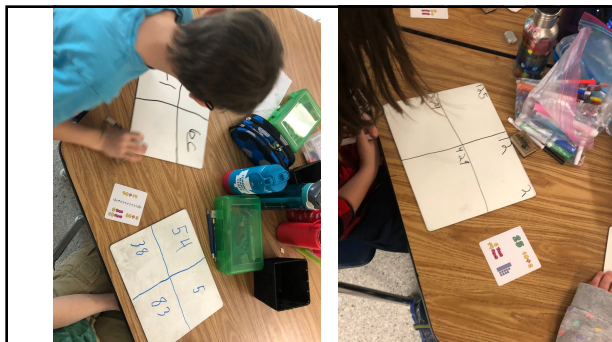
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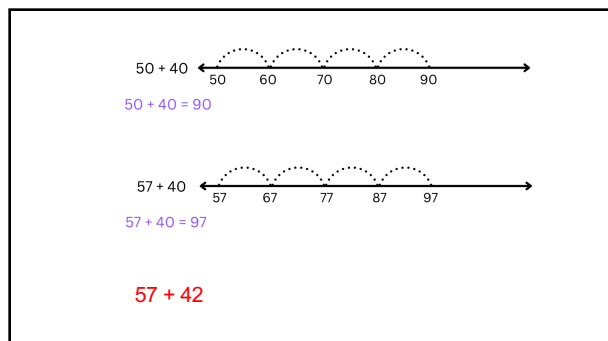
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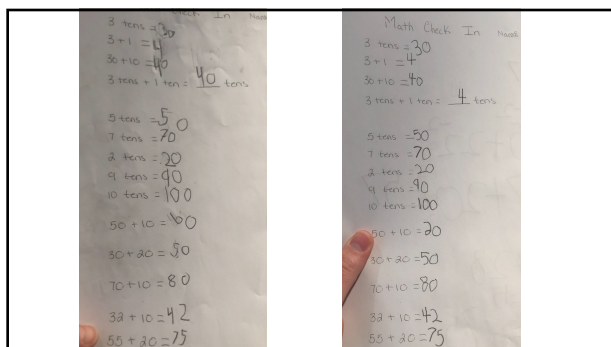
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Solve Multi-Digit ADDITION PROBLEMS

Strategies and models are not the same thing when solving a math problem. When we solve problems mentally, we need a way to show others how we solved the problem which we do through models.

A **strategy** is how you solve the problem.

A **model** is how you show the problem or your strategy.

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"Teachers often **confuse tools with strategies**. Unifix cubes or base ten blocks or paper and pencil are not different strategies. They are different tools. **Representing** the problem with stacks of Unifix cubes grouped into tens, or with base ten blocks, or by writing "10" seven times are all the **same mathematically**. No benefit is derived by changing tools unless the new tool helps the child develop a **higher level of schematizing**..."

Fosnot and Dolk,
Young Mathematicians at Work: Constructing
Number Sense, Addition, and Subtraction, p. 21

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ADDITION

What is Addition?

Addition is the process of combining two or more things together.

$$\text{Addend} + \text{Addend} = \text{Sum}$$

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What are the Common Properties of Addition?

COMMUTATIVE

The order of the addends **does not** matter when finding the sum.

If you count 5 buttons first, then three buttons it is **the same amount** as if you counted 3 buttons first then five buttons.

Ex) $5 + 3 = 3 + 5$

ASSOCIATIVE

The sum is the same regardless how the addends are grouped.

If you add 4 blocks and two blocks together, then add one more you get the same amount as if you add 2 blocks and 1 block together, then add 4 more blocks.

Ex) $(4 + 2) + 1 = 4 + (2 + 1)$

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Which strategies have legs?

Count On

Standard / Expanded Algorithm

Partial Addends / Place Value

Making Tens

Doubles / Near Doubles

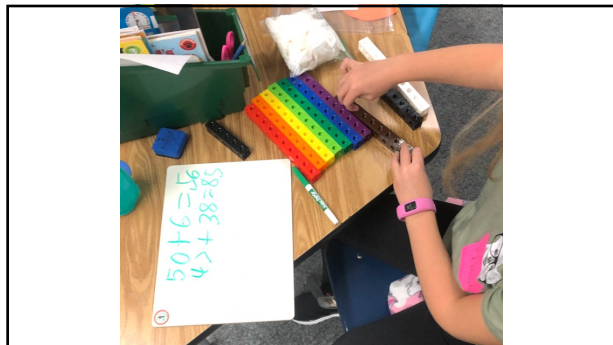
Break Apart Only One Number

Friendly Numbers

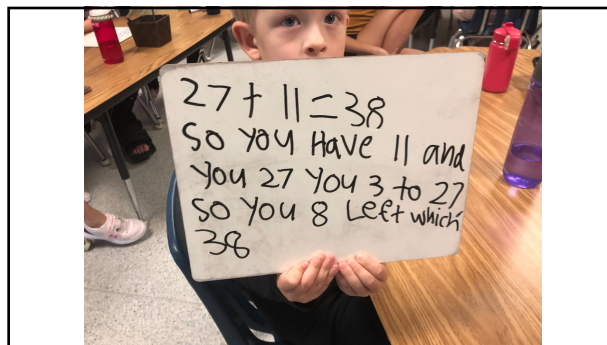
Known Facts

Count All 3 Times

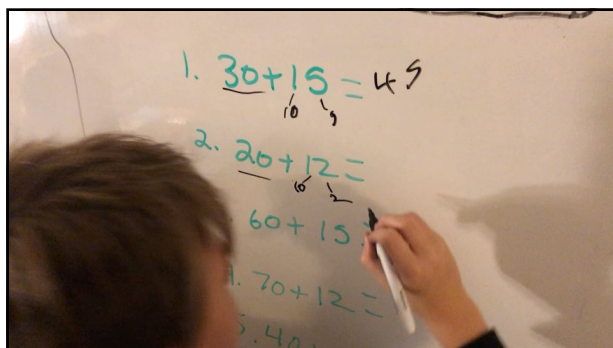
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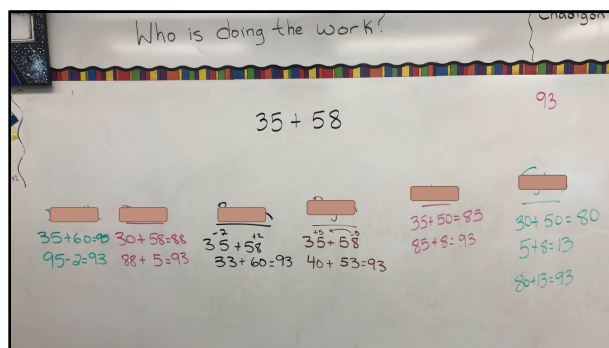
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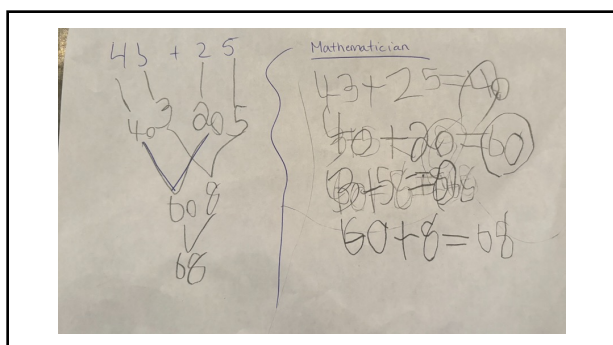
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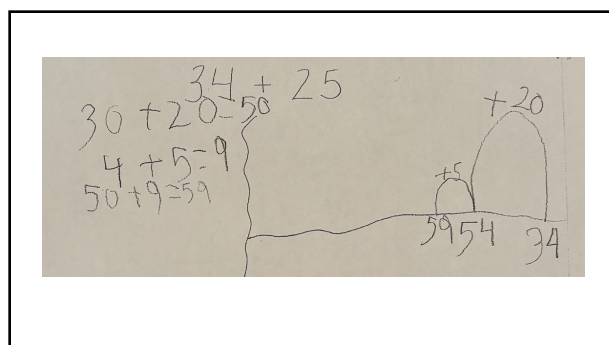
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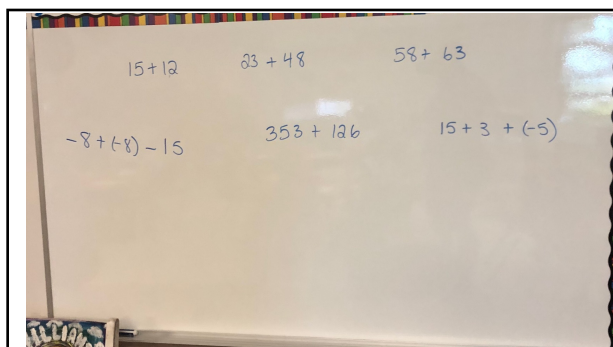
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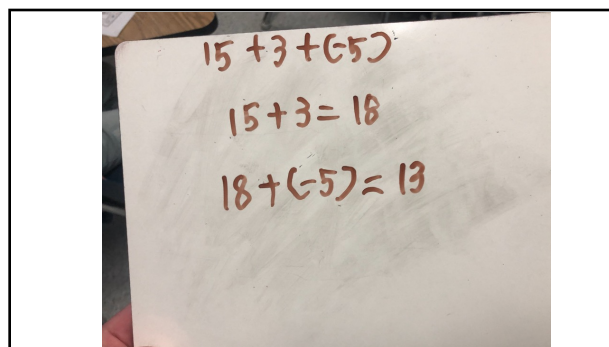
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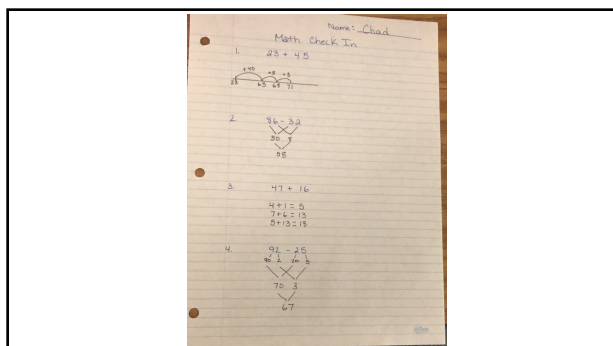
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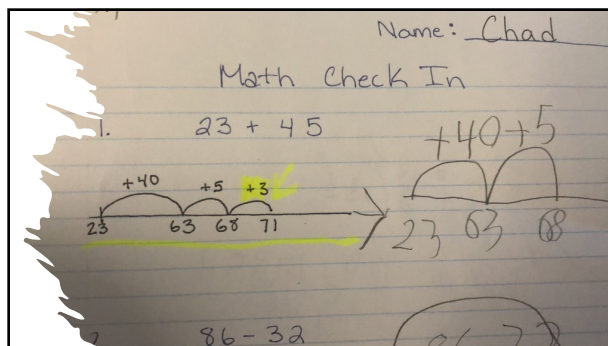
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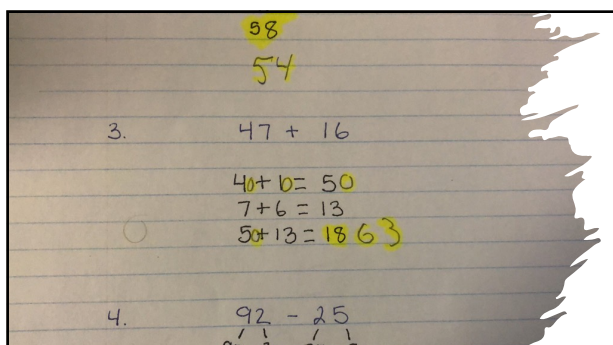
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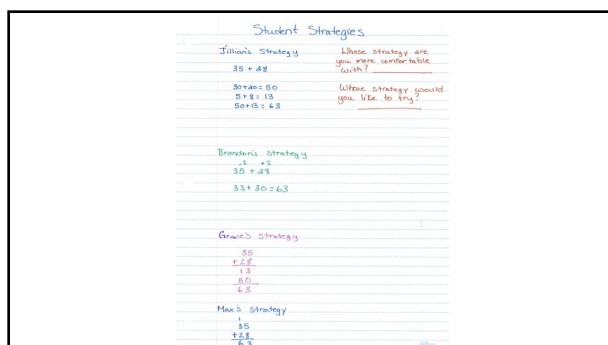
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SUBTRACTION

What is Subtraction?

Subtraction is the "inverse" of addition and it undoes what addition accomplishes. Subtraction is finding the difference between two quantities. Subtraction can be interpreted the following ways: 1) take away, 2) comparison, and 3) the distance between two thing 4) missing addend (addition).

Minuend - Subtrahend = Difference

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What are the Common Properties of Subtraction?

COMMUTATIVE	ASSOCIATIVE
The order does matter when you subtract.	The difference is not the same regardless how the quantities are grouped.
If you have 5 buttons and remove 3 buttons it is not the same amount as if you had 3 buttons and removed 5 buttons.	Ex) $(4 - 2) - 1 \neq 4 - (2 - 1)$ $(3) - 1 \neq 4 - (1)$ $2 \neq 3$
Ex) $5 - 3 \neq 3 - 5$	

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Which strategies have legs?

Count Back	Standard / Expanded Algorithm	
	Constant Difference	
Place Value		Negative Numbers
Adding Up		Adjusting One Number

73

Who is doing the work? Chadigan!

$84 - 37$

Chad

 $84 - 40 = 44$
 $44 + 3 = 47$

$70 - 30 = 40$
 $14 - 7 = 7$
 $40 + 7 = 47$

$84 - 37$
 $87 - 40 = 47$

$84 - 30 = 54$
 $54 - 7 = 47$

$80 - 30 = 50$
 $4 - 7 = -3$
 $50 + (-3) = 47$

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- ① $63 - 28$
- ② $234 - 127$
- ③ $91 - 46$
- ④ $523 - 278$

75

Handwritten notes on a piece of paper:

- $63 - 28$
- $63 - 8 = 55$
- $55 - 20 = 35$
- $63 - 28 = 35$
- $65 - 30 = 35$
- $30 + 5 = 35$
- $63 - 30 = 33$
- $33 + 2 = 35$
- $60 - 20 = 40$
- $3 - 8 = -5$
- $40 + (-5) = 35$
- $93 - 28$
- $90 - 20 = 70$
- $3 - 8 = -5$
- $70 + (-5) = 65$

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Handwritten notes on a piece of paper:

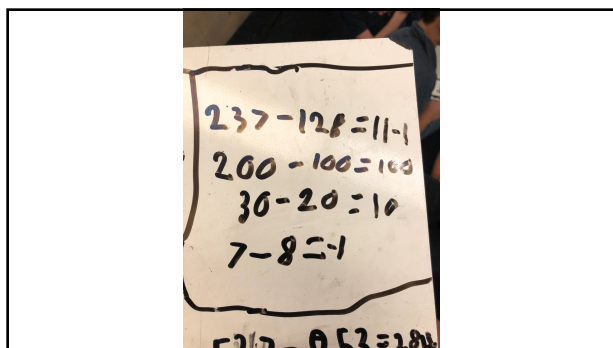
- $234 - 127$
- $234 - 120 = 114$
- $114 - 7 = 107$
- $230 - 120 = 110$
- $14 - 7 = 7$
- $100 + 7 = 107$
- $200 - 100 = 100$
- $34 - 27 = 7$
- $100 + 7 = 107$

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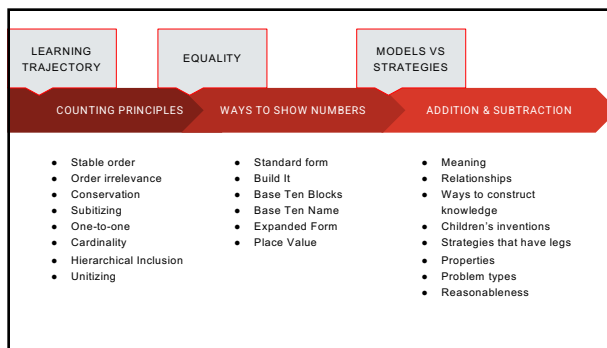
Handwritten on a whiteboard:

$62 - 27$

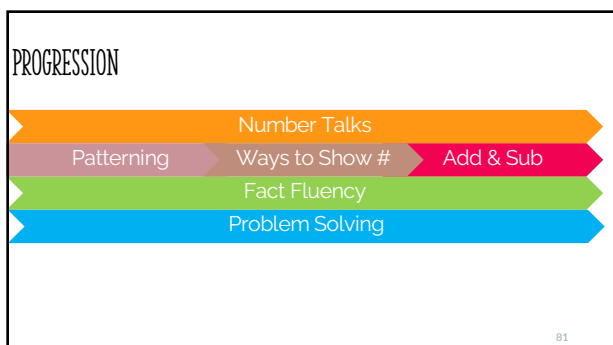
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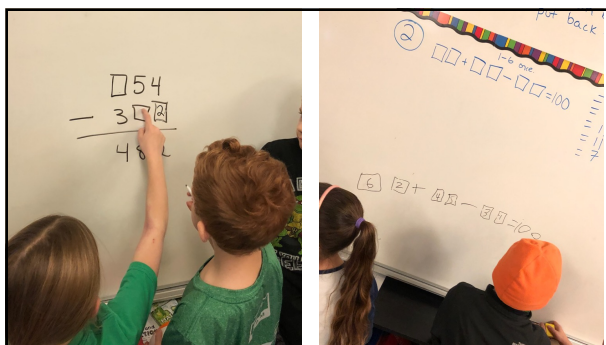
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81



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Text message conversation about a math problem:

Chad Williams: Ss wanted to continue working on this problem at home and engage with their families as they solve it. I sent an email to families and it has been great to hear their replies. Knowing where ur Ss are at doesn't always mean and in isolation.

Note to self: Hey, Students wanted me to send this mat we were working on today in class. Use the numbers 1 to 6 only once to 3 digit numbers that add up to 500 or as close as you can.

142+356=498
152+346=498

This was as close as we got tonight, the fun!

USE numbers 1-6 only once
1 2 3 4 5 6
 $\square\square\square + \square\square\square = 500$

Use the digits 1-6 only once to come up with two 3-digit numbers that add up to 500 or as close as you can.

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Email conversation about a math problem:

Sent: Monday, September 23, 2019 8:17:00 PM

To: Chad Williams
<chad.williams@spirited.ca>

Subject: Re: Math Problem from today.

The question is.. is this actually possible?

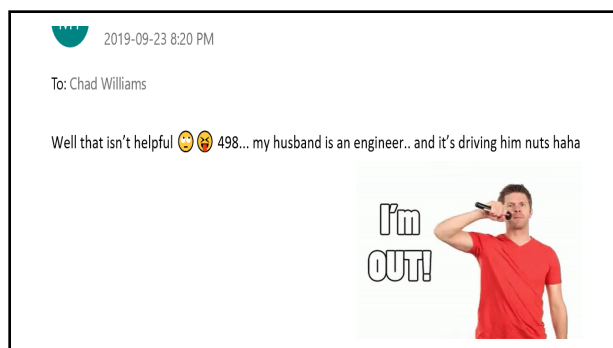
On Mon, Sep 23, 2019 at 8:18 PM Chad Williams <chad.williams@spirited.ca> wrote:

Hmm.. Good question, might be worth trying to figure out why it doesn't work then... lol

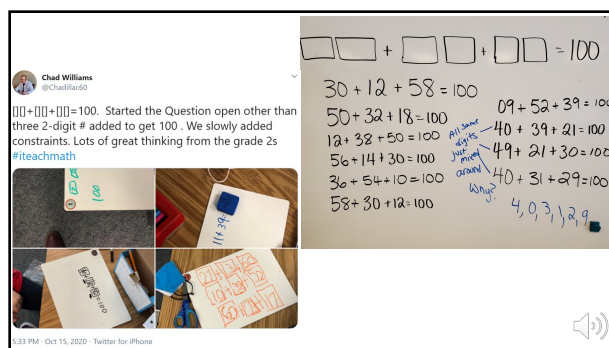
If it doesn't work then how close can you get to 500.

Chad Williams
Clavet Composite School

84



85



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Numbers Menu Task:

Build as *few* of numbers with base ten blocks as possible to satisfy each constraint at least once.

A.	Use more than 10 base ten blocks	B.	The number is less than 50
C.	Includes one of each hundred flat, ten rod and one unit.	D.	The number is greater than 100
E.	Has more ten rods than one units.	F.	Contains exactly 7 one units

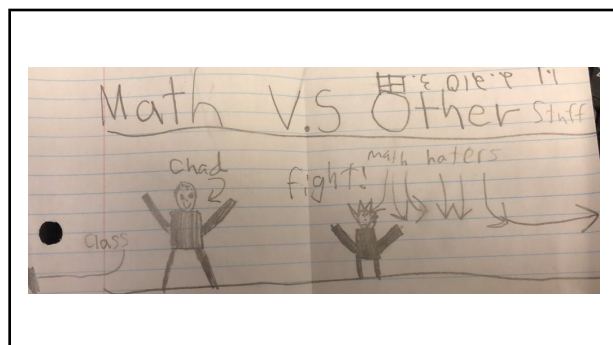
Which constraints pair nicely?
 Which constraints cannot be paired?
 Is it possible to solve in 2, 3, or 4 numbers?

Describe how and why you built each numbers.
 Be sure to identify which groups of numbers satisfy which constraints.

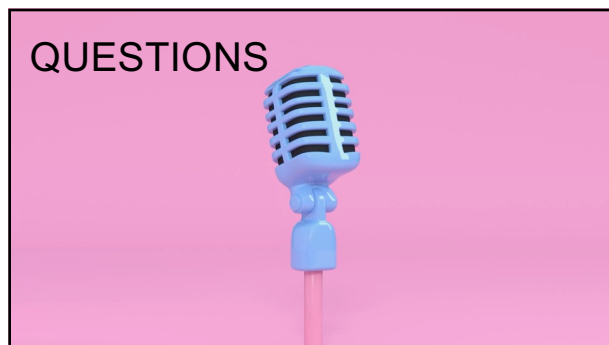
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Thank You

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