MULIPLICATION

What is Multiplication?

Multiplication is used in many situations. 1) Repeated addition, 2) Equal groups or sets, 3) Area, 4) Rate/Comparison, 5) Combinations

Factor × Factor = Product

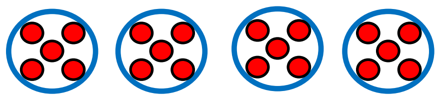
Repeated Addition

The first factor (4), tells us how many time to add the second factor, 5.

$$4 \times 5 = 5 + 5 + 5 + 5$$

Equal Groups

4×5 is the total number of objects.

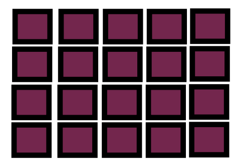


4 sets of 5

Array

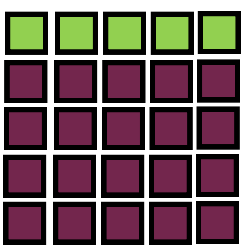
4×5 is the total number of items in a 4-by-5 array.

4 rows with 5 counters
in each has 20 counters altogether.



Rate/Comparison

4×5 represents the final amount when a 5-unit measure is increased to 4 times its original value.



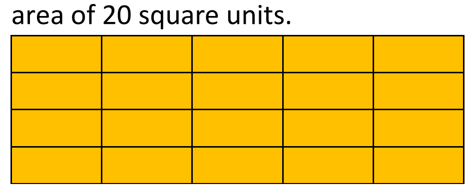
Combinations

4×5 is the number of ice cream flavours to cone combinations. If there are 4 flavours of ice cream and 5 cones.

Area

4×5 is the area of a 4-by-5 rectangle

4 units wide by 5 units long has an area of 20 square units.



Where is Multiplication in the Sask Curriculum?

Grade Three

Demonstrate an understanding of multiplication of whole numbers from 1×1 to 5×5.

Grade Four

Demonstrate an understanding of multiplication of:

- whole numbers less than 100
- 2- or 3-digit factors multiplied by 1-digit factor. *Ex)* 23×6

Grade Five

Demonstrate an understanding of multiplication up to 2-digit by 2-digit whole numbers.

Grade Six

Demonstrate an understanding of multiplication of decimals by 1-digit whole number factor.

What are the Common Properties of Multiplication?

COMMUTATIVE

The order of the factors **does not** matter when finding the product.

If you have 4 groups of 3, it **is the same amount** as if you have 3 groups of 4.

Ex) $4 \times 3 = 3 \times 4$

ASSOCIATIVE

The product is the same regardless how the factors are grouped.

Each marble costs $1. There are 2 marbles in each pouch. How much will it cost if we need to buy 3 pouches? We can find out the cost for each pouch and then the total cost **or** we can find out how many marbles in total and then the total cost.

Ex) $(1 \times 2) \times 3 = 1 \times (2 \times 3)$

DISTRIBUTIVE

Either one of the two factors in a product can be split (decomposed) into two or more parts, then each multiplied separately and then added.

If you have 3 groups of 4 and another 3 groups of 4 **it is the same** as 3 groups of 8.

Ex) $3 \times 8 = (3 \times 4) + (3 \times 4)$

ZERO & INDENTITY

When we multiply a factor by zero the product will be zero.

Ex) $3 \text{ x } 0 = 0$

When we multiply a number by 1 the answer is the number you started with.

Ex) $5 \times 1 = 5$

Strategies vs Models

Strategies and models are not the same thing when solving a math problem. When we solve problems mentally, we need a way to show others how we solved the problem which we do through models.

A **strategy** is how you solve the problem.

A **model** is how you show the problem or your strategy.

For example, I may use a landmark strategy (p.3) to multiply two numbers mentally and model my strategy using an area model.

What are the Common Strategies of Multiplication?

The order and sequence of the following strategies is not how they should be introduced or instructed to children. Sharing the various strategies will help you identify the method in which your child might be solving problems in order to engage in conversation with your child confidently and comfortably about their strategy.

The multiplication strategies below should be taught through the Concrete - Representational – Abstract approach, which allows students to build conceptual understanding first through concrete manipulatives, then drawings and representations and finally with abstract numbers. Skipping these steps and moving quickly to rote memorization will result in students having procedural understanding of multiplication which may result in coming to the correct answer, however the student will be unable to be flexible and efficient in transferring their understanding to other problems.

There is no expectation that your child will use or learn all the strategies below but rather should be exposed to a variety of strategies that they understand and can use depending on the situation.

Single Digit Multiplication Strategies

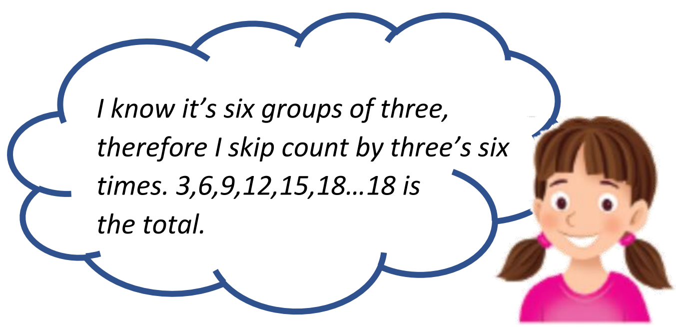
Repeated Addition / Skip Counting

This strategy is often used with students who are starting to learn multiplication. Students model multiplication with equal sets/groups which they add or skip count to come to the total (product).

Example: 6 × 3

6 groups of 3.

3 + 3 + 3 + 3 + 3 + 3 = 18

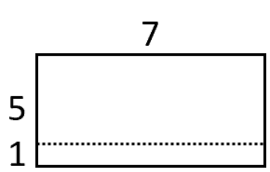


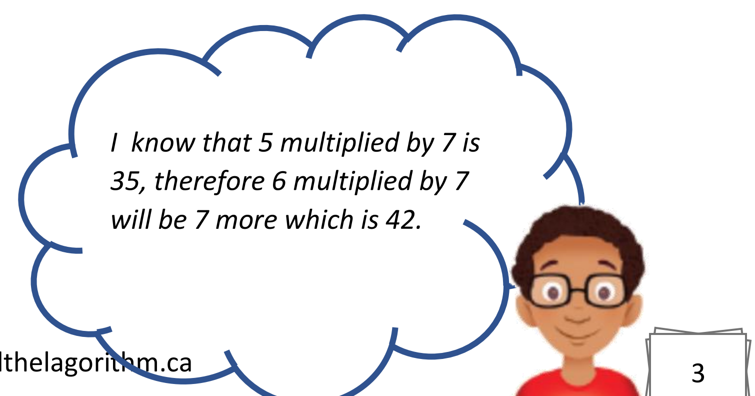
Landmark / Friendly Numbers / Compensation

Students change one of the factors to create an easier multiplication problem. The factors students select, and change will depend on what factors they find friendly. Students use their experience and knowledge of known facts that they can recall to help them solve other multiplication questions.

Example: 6 × 7

<u>-1</u> (group of 7)
5 × 7 = 35
+ 1 group of 7 = 42





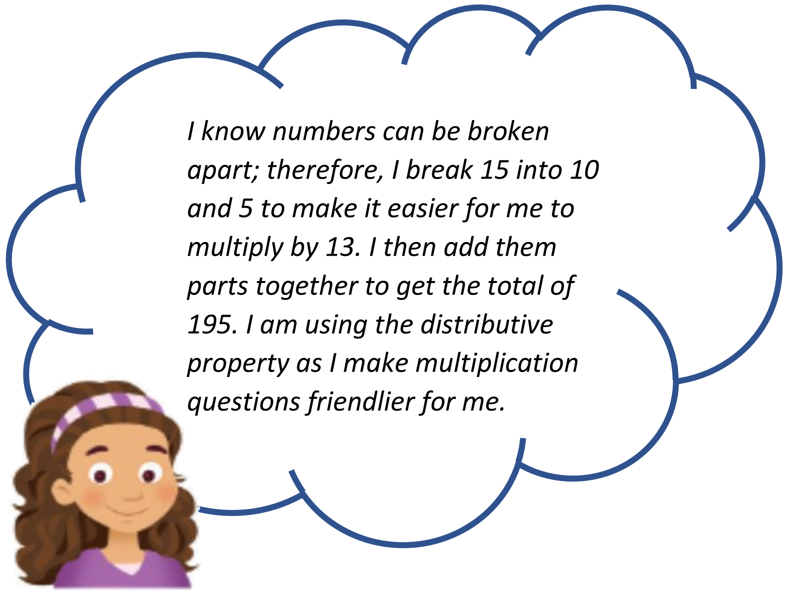
Multi-Digit Multiplication Strategies

Partial Products

Students decompose or break apart factors into addends allowing them to use smaller problems to solve larger and more difficult problems.

Example 1: 13 × 15

$13 \times (10 + 5)$
$13 \times 10 = 130$
$13 \times 5 = 65$
$130 + 65 = 195$



Example: 24 × 17

$(20 + 4) \times (10 + 7)$
$20 \times 10 = 200$
$20 \times 7 = 140$
$4 \times 10 = 40$
$4 \times 7 = 28$
$200 + 140 + 40 + 28 = 408$

AREA MODEL

Explicitly represents what is occurring throughout the strategy of partial products.

	20	4
10	**200**	**40**
7	**140**	**28**

BOX METHOD

A short form method of the area model that indicates the multiplication of the partial products.

	20	4
10	200	40
7	140	28

Doubling and Halving

This strategy is used to change the problem into a friendly problem by halving one factor and doubling the other factor. Students will notice this relationship when they begin to build arrays that have the same area and study the patterns of the dimensions.

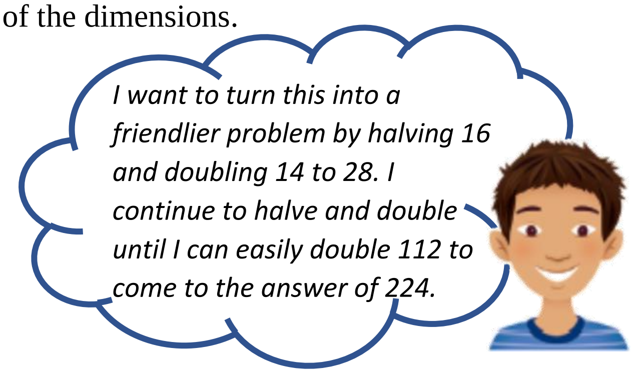
Example: 16 × 14

16×14

8×28

4×56

$2 \times 112 = 224$



Multiplication

Breaking Factors into Smaller Factors

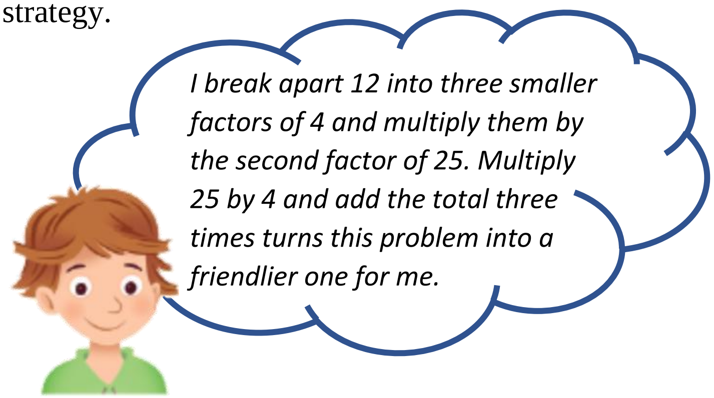
Students break factors into smaller factors to create easier problems and an efficient strategy. The associative property is at the core of this strategy.

Example: 12×25

$(4 \times 25) + (4 \times 25) + (4 \times 25)$

$100 + 100 + 100 = 300$

Recall associative property:
$(3 \times 4) \times 25 = 3 \times (4 \times 25)$

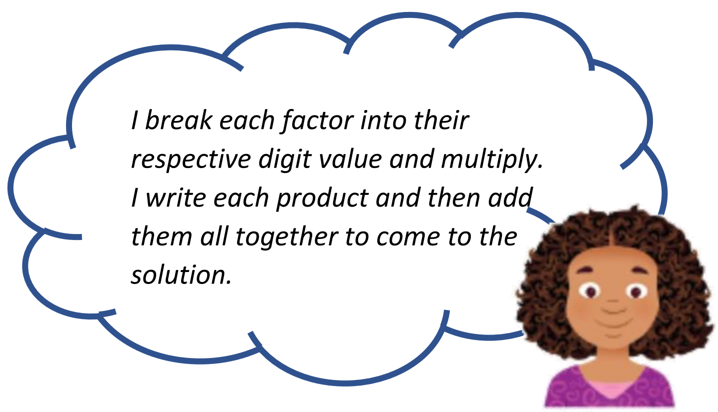


Expanded Algorithm

This strategy is similar Partial Products, except it is written in a different way as students move to a more efficient way of writing their thinking. This strategy explicitly shows that students understand place value of each digit in the factors.

Example: 34×18

$$\begin{array}{r} 34 \\ \times 18 \\ \hline 32 \\ 240 \\ 40 \\ 300 \\ \hline 612 \end{array}$$



Standard Algorithm

Students work from the right to the left multiplying the parts of each factor and efficiently notes what they are doing. Students may turn multi-digit multiplication into single digit multiplication for each place value amount. The difference here is that students understand what they are doing throughout the algorithm as they have come to this strategy with understanding as they constructed their knowledge of other strategies. When asked to explain their strategy, they can explain using correct language and understanding.

Example: 34×18

$$\begin{array}{r} {}^{3} \\ 3\;4 \\ \times\;1\;8 \\ \hline {}^{1}2\;7\;2 \\ 3\;4\;0 \\ \hline 6\;1\;2 \end{array}$$