SUBTRACTION

"It is very hard for us adults to not equate familiarity with understanding. We are very familiar with it [standard algorithm], therefore we feel we understand it, but imagine teaching it for the very first time, that is very, very hard"

James Tanton, Mathematician

What is Subtraction?

Subtraction is the "inverse" of addition and it undoes what addition accomplishes. Subtraction is finding the difference between two quantities. Subtraction can be interpreted the following ways: 1) take away, 2) comparison, and 3) the distance between two things or missing addend.

Minuend - Subtrahend = Difference

Saskatchewan CURRICULUM
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Grade One

Demonstrate an understanding of subtraction of whole numbers under 20.

Grade Two

Demonstrate an understanding of subtraction (limited to 1 and 2-digit numbers) of whole numbers under 100.

Grade Three

Demonstrate an understanding of addition of whole numbers (limited to 1, 2, and 3-digit numbers) under 1000.

Grade Four

Demonstrate an understanding of subtraction of whole numbers (limited to 3 and 4-digit numbers) under 10 000.

Grade Five

Demonstrate an understanding of subtraction with decimals to the thousandths.

Grade Six

Demonstrate understanding of the order of operation on whole numbers (excluding exponents) with and without technology.

What are the Common Properties of Subtraction?

COMMUTATIVE

The order **does** matter when you subtract.

If you have 5 buttons and remove 3 buttons **it is not the same** amount as if you had 3 buttons and removed 5 buttons.

Ex) $5 - 3 \neq 3 - 5$

ASSOSCIATIVE

The difference **is not the same** regardless how the quantities are grouped.

Ex) $(4 - 2) - 1 \neq 4 - (2 - 1)$

$(3) - 1 \neq 4 - (1)$

$2 \neq 3$

Strategies Vs Models

Strategies and models are not the same thing when solving a math problem. When we solve problems mentally, we need a way to show others how we solved the problem which we do through models.

A **strategy** is how you solve the problem.

A **model** is how you show the problem or your strategy.

For example, I may use a chunking strategy (see below) to add two numbers mentally and model my strategy on an open number line.

What are the Common Strategies of Subtraction?

The order and sequence of the following strategies is not how they should be introduced or instructed to children. Sharing the various strategies will help you identify the method in which your child might be solving problems in order to engage in conversation with your child confidently and comfortably about their strategy.

The subtraction strategies below should be taught through the Concrete - Representational – Abstract approach, which allows students to build conceptual understanding first through concrete manipulatives, then drawings and representations and finally with abstract numbers. Skipping these steps and moving quickly to rote memorization will result in students having procedural understanding of subtraction which may result in coming to the correct answer, however the student will be unable to be flexible and efficient in transferring their understanding to other problems.

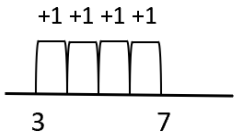
There is no expectation that your child will use or learn all the strategies below but rather should be exposed to a variety of strategies that they understand and can use depending on the situation.

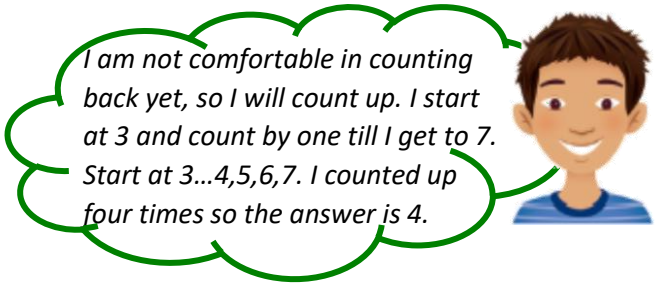
Single Digit Subtraction

Adding Up

Students build on their strength of addition and add up from the subtrahend the number being subtracted) to the minuend (the whole) to find the difference between the two numbers. Students will jump to the nearest ten or friendly numbers and the larger the jumps the more efficient the strategy will be.

Example: 7 – 3 = 4

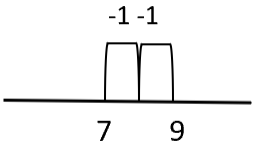


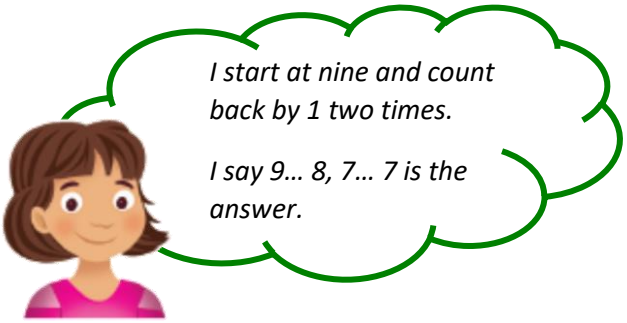


Removal / Counting Back

Students can decompose the subtrahend into easy to remove parts and take these jumps backwards from the minuend. The bigger the jumps the more efficient the strategy will be.

Example: 9 – 2 = 7



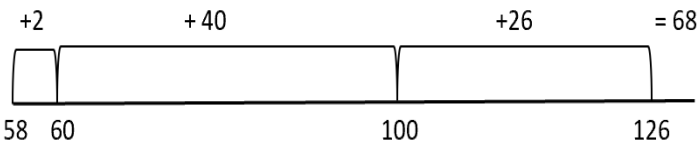


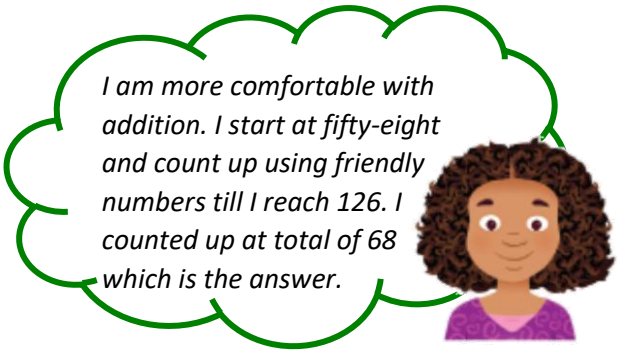
Multi-Digit Subtraction

Adding Up

Students build on their strength of addition and add up from the subtrahend the number being subtracted) to the minuend (the whole) to find the difference between the two numbers. Students will jump to the nearest ten or friendly numbers and the larger the jumps the more efficient the strategy will be.

Example:126-58



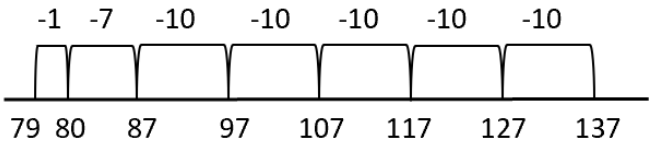


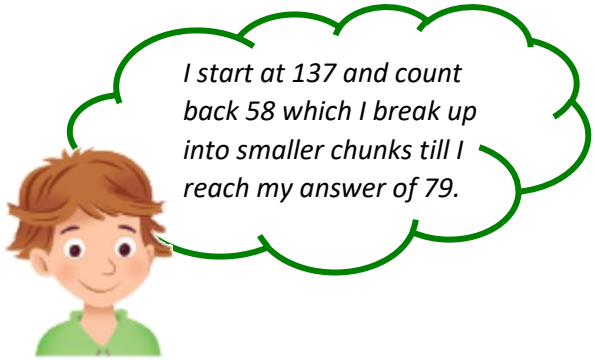
Removal / Counting Back

Students can decompose the subtrahend into easy to remove parts and take these jumps backwards from the minuend. The bigger the jumps the more efficient the strategy will be.

Example:137 – 58

137 – (10 + 10 + 10 + 10 + 10 + 7 + 1)





Place Value & Negative Numbers

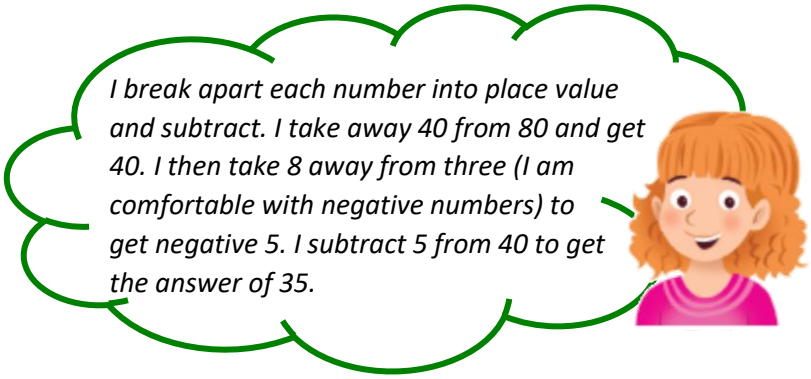
Students break each number into place value. Like place values are grouped and then subtracted, negative values might result.

Example:83 – 48

(80 + 3) – (40 – 8)

	80	3
–	40	8
	40	−5

$40 - 5 = 35$

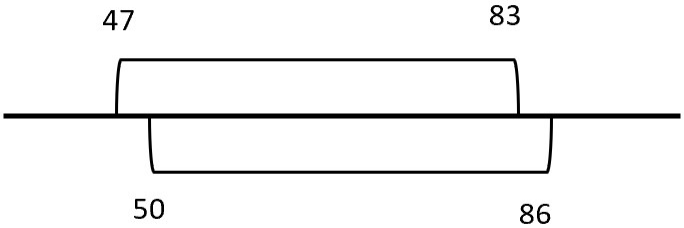


Constant Difference

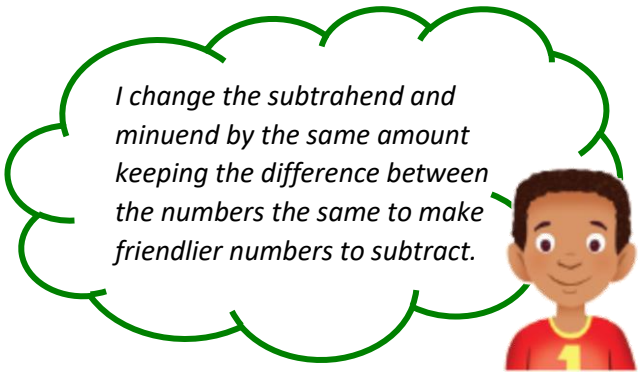
As students become more familiar with subtraction and begin to understand subtractions as the difference between quantities, they can add or subtract the same amount from both numbers and keep the same difference. Manipulating the numbers allows students to create friendlier problems.

Example: 83 – 47

47 + 3 = 50 83 + 3 = 86



$86 - 50 = 36$

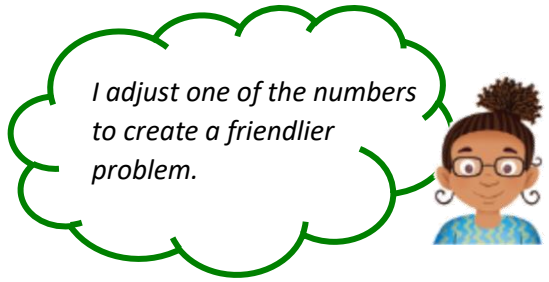


Adjusting One Number

Adjusting one of the numbers can turn the subtraction problem into a friendlier one. The decision is what number you are going to adjust to create a friendlier problem. The student's understanding of part/whole relationships will help them reason through these decisions.

Example: $164 - 28$

$$\begin{array}{r} \overset{+2}{} \\ 164 - 30 = 134 \\ \underset{+2}{} \\ = 136 \end{array}$$

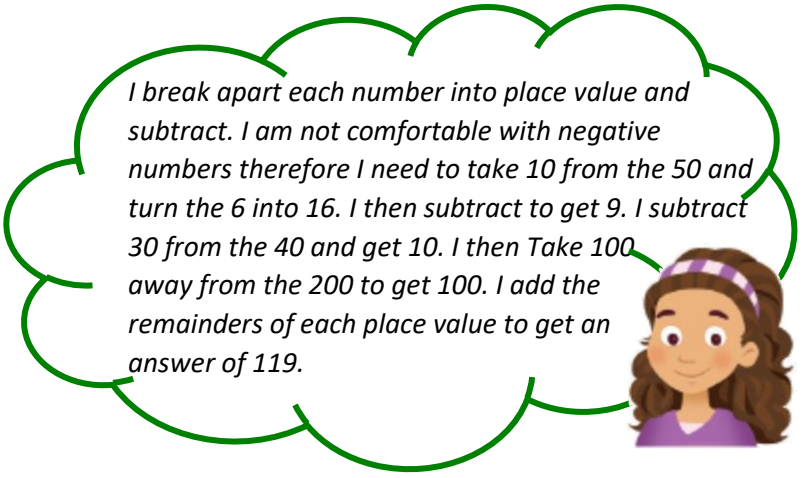


Expanded Algorithm 1

This strategy is like Place Value, except it is written in a different way as students move to a more efficient way of writing their thinking. This strategy explicitly shows that students understand place value of each digit in the minuend and subtrahend.

Example: 256 – 137

$$\begin{array}{rrrr} & 200 & + \overset{40}{\cancel{50}} & + \overset{1}{6} \\ - & 100 & + 30 & + 7 \\ \hline & 100 & + 10 & + 9 \end{array}$$

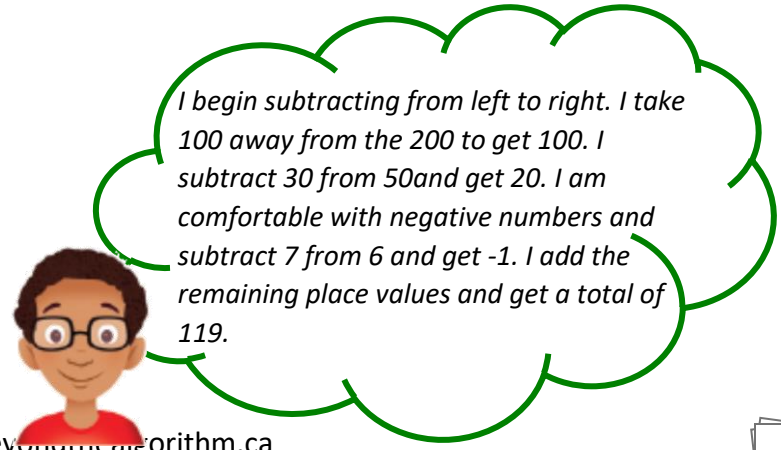


Expanded Algorithm 2

Students subtract like place value amounts. This strategy explicitly shows the subtraction of each like place value. Students then add up the remaining amounts and the ways they add them up may differ depending on amounts and understanding. Students who understand and are comfortable using negative numbers will be more efficient than those who must ungroup.

Example: 256 – 137

$$\begin{array}{rrrr} & 2 & 5 & 6 \\ - & 1 & 3 & 7 \\ \hline & 1 & 0 & 0 \\ & & 2 & 0 \\ & & & -1 \\ \hline & 1 & 1 & 9 \end{array}$$



Standard Algorithm

Students work from the right to the left subtracting each like place value amount and efficiently notes what they are doing. Students may turn multi-digit subtraction into single digit subtraction for each place value amount. The difference here is that students understand what they are doing throughout the algorithm as they have come to this strategy with understanding as they constructed their knowledge of other strategies. When asked to explain their strategy, they can explain using correct language and understanding.

Example:256 – 137

$$\begin{array}{r} {}^{}{}^{4}{}^{} \\ 2\ \cancel{5}\ {}^{1}6 \\ -\ 1\ 3\ 7 \\ \hline 1\ 1\ 9 \end{array}$$

I will work from right to left. need to take 10 from the 50 and turn the 6 into 16. I then subtract to get 9. I subtract 30 from the 40 and get 10. I then Take 100 away from the 200 to get 100. I add the remainders of each place value to get an answer of 119.

I want to reemphasize that the standard algorithm is not the be it end all strategy we are hoping students get to solve all math problems. We want students to be able to use an efficient personal strategy that lends itself to the problem that is presented.

Reflection

Think about how you would solve this problem and how you would hope your child solves the following problem. Would it be an algorithm? Or having the number sense to solve it in a more efficient way?

1000 – 998 =